# All regular graphs of small odd order are vertex-magic 

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#### Abstract

This paper deals with vertex-magic total labellings of graphs. Earlier work by many authors has shown many infinite families of graphs to admit such labelings. The fact that many of these graphs are regular led MacDougall to conjecture that all non-trivial regular graphs are vertexmagic. Previously Gray and MacDougall showed that all odd-order $r$ regular graphs $(r \geq 2)$ of order up to $v=19$ are vertex-magic. In this paper, we report on computations that extend this range, to show that all odd-order $r$-regular graphs $(r \geq 2)$ of order up to $v=29$ are vertexmagic.


## 1 Introduction

A vertex-magic total labeling (VMTL) on a graph $G(V, E)$ with $v$ vertices and $e$ edges is a one-to-one mapping $\lambda$ from the vertices and edges onto the integers $1,2, \ldots, v+e$ so that the sum of the label on a vertex and the labels of its incident edges is constant, independent of the choice of vertex. This sum is called the magic constant and varies depending on which labels are assigned to the vertices and which to the edges. The second author has conjectured [4]

Conjecture 1 With the exception of $K_{2}$ and $2 K_{3}$, all regular graphs have at least one VMTL.

Based on McQuillan's work [6], Gray (in [2]) provided a major step toward the solution of this conjecture. He developed a procedure for beginning with a graph possessing a VMTL and adjoining an arbitrary 2 -factor to produce a graph of the same order but larger size which also possesses a VMTL. Using that procedure, Gray and MacDougall showed [3] that for $r \geq 4$ every $r$-regular graph of odd order $v \leq 17$ possesses a VMTL. In this paper we report the results of computations that extend this result to orders $v \leq 29$.

## 2 Strong VMTLs

Gray's method applies to those labelings which are strong, i.e. have the $v$ largest labels assigned to the vertices (and thus the smallest possible magic constant). In [3] it is shown that the range of feasible values for the magic constant $k$ of an $r$-regular graph is determined by

$$
v r^{2}+2(v+1)(r+1) \leq 4 k \leq v r^{2}+2(v+1)(r+1)+2 v r
$$

and for the labeling to be strong we must have $k=\frac{1}{4}\left(v r^{2}+2(v+1)(r+1)\right)$. This will only be possible if $v$ is odd and $r$ is even, or if $v$ is a multiple of 4 and $r$ is odd. Thus, for example, even cycles do not have strong VMTLs but odd cycles do (the standard caterpillar labeling is an example).

Theorem 1 ([2, Theorem 2.1]) If $G$ is a graph with a spanning subgraph $H$ which possesses a strong VMTL and $G-E(H)$ is even-regular, then $G$ also possesses a strong VMTL.

The power of this theorem is illustrated by the case where the starting graph $H$ is an odd cycle. Every odd cycle admits a strong VMTL and adjoining 2-factors repeatedly gives us the following result.

Corollary 1 ([2]) Every Hamiltonian regular graph of odd order possesses a strong VMTL.

## 3 Strong VMTLs of 2-regular Graphs

While it is true that asymptotically almost all regular graphs possess a spanning cycle (so that the corollary above applies), we must consider those that do not. Part of the investigation in [3] depended on the classical result of Petersen that every even-regular graph has a 2-regular spanning subgraph, in other words, is spanned by a disjoint union of cycles. Thus to prove that all even-regular graphs of odd order $v$ are strongly vertex-magic, it is enough to show that all 2-regular graphs of order $v$ have strong VMTLs. Table 1 below shows the number $\gamma$ of 2 -regular graphs for the relevant values of $v$; these numbers appear as the odd terms in sequence A008483 in Sloane's On-Line Encyclopedia of Integer Sequences [9].

Table 1: The number of 2-regular graphs of order $v$ and the strong magic constant $k$

| $v$ | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | 33 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | 9 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 69 | 74 | 79 | 84 |
| $\gamma$ | 1 | 1 | 2 | 4 | 6 | 10 | 17 | 25 | 39 | 60 | 88 | 130 | 191 | 273 | 391 | 556 |

Some examples of strong VMTLs for 2-regular graphs are shown in Figures 1 and 2: those in the first figure are disjoint unions of distinct cycles, while the graph in
the second figure has a repeated cycle. Not every odd-order 2-regular graph has a strong VMTL. It was noted in [3] that $C_{3} \cup C_{4}$ with order $7,2 C_{3} \cup C_{5}$ with order 11 and $3 C_{3} \cup C_{4}$ with order 13 had no strong VMTL. This led the authors of [3] to wonder whether these were the initial examples in two infinite families of such graphs. Constructions by McQuillan [7] and our own computations now suggest that in fact these are the only counterexamples.

Figure 1: Two Examples of Strong VMTLs








Figure 2: Strong VMTL of $7 C_{3} \cup C_{6}$








The tables presented in this paper contain examples of a strong VMTL for all the other 2-regular graphs of orders up to 29 . These were discovered by extensive computer searches. In all the tables the graphs are listed in reverse lexicographic order on the set of cycle lengths. The examples in these tables thus constitute a proof of the following result:

Theorem 2 Other than the three graphs, $C_{3} \cup C_{4}, 2 C_{3} \cup C_{5}$ and $3 C_{3} \cup C_{4}$, every odd-order 2 -regular graph of odd order less than 30 has a strong vertex magic total labelling.

Proof. Tables 2, 3 and 9 to 22 list the edge label cycles of a strong VMTL for each graph other than the three exceptions.

Table 2: Strong VMTLs of the 2-regular graphs of odd order $\leq 13$

| $v$ | $k$ | Graph | Edge Labels |
| :---: | :---: | :---: | :---: |
| 3 | 9 | $\mathrm{C}_{3}$ | (1,3,2) |
| 5 | 14 | $\mathrm{C}_{5}$ | (1,4,2,5,3) |
| 7 | 19 | $\begin{array}{r} C_{7} \\ C_{3} \cup C_{4} \\ \hline \end{array}$ | $\begin{aligned} & (1,5,2,6,3,7,4) \\ & \text { no strong VMTL } \end{aligned}$ |
| 9 | 24 | $\begin{array}{r} C_{9} \\ C_{3} \cup C_{6} \\ C_{4} \cup C_{5} \\ 3 C_{3} \\ \hline \end{array}$ | $\begin{aligned} & \hline(1,6,2,4,7,3,9,5,8) \\ & (1,7,2,8,6,5)(3,9,4) \\ & (1,7,2,5)(3,8,6,4,9) \\ & (1,9,5)(2,7,6)(3,8,4) \end{aligned}$ |
| 11 | 29 | $\begin{array}{r} C_{11} \\ C_{3} \cup C_{8} \\ C_{4} \cup C_{7} \\ C_{5} \cup C_{6} \\ 2 C_{3} \cup C_{5} \\ C_{3} \cup 2 C_{4} \\ \hline \end{array}$ | $\begin{aligned} & \hline(1,7,2,5,10,6,11,3,8,4,9) \\ & (1,9,8)(2,6,7,4,3,11,5,10) \\ & (1,10,6,7)(2,5,4,11,3,9,8) \\ & (1,10,7,6,2,8)(3,9,5,11,4) \\ & \text { no strong VMTL } \\ & (1,10,3,11)(2,6,4,5)(7,9,8) \\ & \hline \end{aligned}$ |
| 13 | 34 | $\begin{array}{r} C_{13} \\ C_{3} \cup C_{10} \\ C_{4} \cup C_{9} \\ C_{5} \cup C_{8} \\ C_{6} \cup C_{7} \\ 2 C_{3} \cup C_{7} \\ C_{3} \cup C_{4} \cup C_{6} \\ C_{3} \cup 2 C_{5} \\ 2 C_{4} \cup C_{5} \\ 3 C_{3} \cup C_{4} \end{array}$ | $\begin{aligned} & \hline(1,8,12,5,13,2,11,3,9,10,6,4,7) \\ & (1,8,2,12,6,13,4,7,9,11)(3,10,5) \\ & (1,9,8,12)(2,6,13,3,11,7,4,5,10) \\ & (1,10,8,12,5,9,4,11)(2,7,3,13,6) \\ & (1,9,8,11,3,10)(2,6,12,4,5,7,13) \\ & (1,9,10)(2,7,13)(3,5,8,4,12,6,11) \\ & (1,7,10)(2,8,4,5,9,11)(3,12,6,13) \\ & (1,10,4,9,11)(2,7,8)(3,5,12,6,13) \\ & (1,11,3,10)(2,6,9,8)(4,12,7,13,5) \\ & \text { no strong VMTL } \end{aligned}$ |

As a result of the Theorem we can apply Gray's construction, adjoining whatever 2-factors we like to any of these initial 2-regular graphs. Thus we can prove our main result:

Corollary 2 Other than the three graphs $C_{3} \cup C_{4}, 2 C_{3} \cup C_{5}$ and $3 C_{3} \cup C_{4}$, every odd-order regular graph of order less than 30 possesses a strong VMTL.

It was shown in [3] that other than these three exceptional graphs $C_{3} \cup C_{4}, 2 C_{3} \cup C_{5}$ and $3 C_{3} \cup C_{4}$, all even-regular graphs of orders 7,11 and 13 , are vertex-magic. The proof of this relied, in part, on the fact that every quartic graph is spanned by a 2-regular graph other than these three. If we now note that these three exceptions do admit VMTLs (just not strong ones) we obtain the main result of the paper:

Corollary 3 Every odd-order regular graph of order less than 30 is vertex-magic.

Table 3: Strong VMTLs of the 17 2-regular graphs of order 15

| Graph | Edge Labels |
| ---: | :--- |
| $C_{15}$ | $(1,13,9,4,6,14,7,12,11,5,10,2,15,3,8)$ |
| $C_{3} \cup C_{12}$ | $(1,9,2,12,5,4,8,11,7,15,6,14)(3,10,13)$ |
| $C_{4} \cup C_{11}$ | $(1,8,14,6,10,5,7,3,15,4,13)(2,9,12,11)$ |
| $C_{5} \cup C_{10}$ | $(1,14,8,3,7,11,9,5,4,12)(2,10,13,6,15)$ |
| $C_{6} \cup C_{9}$ | $(1,12,11,4,10,2,15,7,9)(3,6,14,5,13,8)$ |
| $2 C_{3} \cup C_{9}$ | $(1,12,11,4,10,8,13,3,9)(2,7,15)(5,14,6)$ |
| $C_{7} \cup C_{8}$ | $(1,10,11,9,14,8,2,13)(3,6,7,5,12,4,15)$ |
| $C_{3} \cup C_{4} \cup C_{8}$ | $(1,10,13)(2,8,14,6,3,15,4,11)(5,12,9,7)$ |
| $C_{3} \cup C_{5} \cup C_{7}$ | $(1,10,11,2,13)(3,6,4,12,7,5,15)(8,9,14)$ |
| $2 C_{4} \cup C_{7}$ | $(1,10,12,9,14,2,13)(3,6,11,7)(4,8,5,15)$ |
| $C_{3} \cup 2 C_{6}$ | $(1,13,5,11,2,9)(3,6,14,7,10,12)(4,8,15)$ |
| $C_{4} \cup C_{5} \cup C_{6}$ | $(1,11,9,13)(2,8,15,4,14,7)(3,10,6,5,12)$ |
| $3 C_{3} \cup C_{6}$ | $(1,11,9)(2,7,14)(3,12,10)(4,13,5,6,8,15)$ |
| $3 C_{5}$ | $(1,8,15,7,14)(2,10,3,11,9)(4,6,13,5,12)$ |
| $2 C_{3} \cup C_{4} \cup C_{5}$ | $(1,8,14)(2,12,11,9)(3,10,7,5,13)(4,6,15)$ |
| $C_{3} \cup 3 C_{4}$ | $(1,10,2,13)(3,7,6,15)(4,5,12)(8,11,9,14)$ |
| $5 C_{3}$ | $(1,12,11)(2,8,14)(3,6,15)(4,13,7)(5,9,10)$ |

## 4 The Number of Strong VMTLs of 2-regular Graphs

Our goal as reported in the last section was to discover a single strong VMTL for each 2-regular graph of odd order. This was enough to prove the existence of a VMTL for every regular graph of odd order $v<31$. However our computations have also provided more convincing evidence supporting Conjecture 1. It turns out that, apart from the small order graphs, there are very large numbers of VMTLs for almost all graphs under consideration. Tables 4 and 5 below contain the results of counting the number of strong VMTLs for the 2-regular graphs (these numbers appear as sequence A176210 in Sloane's OEIS). We found the exact numbers for $v \leq 19$, but the practical limit of complete counting turned out to be at $v=19$, so in Tables 5 and 6 we present lower bounds for $v=21$ and $v=23$. The counts we found are strong evidence that every 2 -regular graph is strongly vertex-magic, and thus that Conjecture 1 is true for even-order graphs. However we still have no good idea how to prove this.

Table 4: The number of strong VMTLs of 2-regular graphs of orders 3 to 17

| A176210 | A177741 |
| ---: | ---: |
| Graph | \#sVMTLs |
| $C_{3}$ | 1 |
| $C_{5}$ | 1 |
| $C_{7}$ | 9 |
| $C_{3} \cup C_{4}$ | $\mathbf{0}$ |
| $C_{9}$ | 31 |
| $C_{3} \cup C_{6}$ | 8 |
| $C_{4} \cup C_{5}$ | 4 |
| $3 C_{3}$ | 2 |
| $C_{11}$ | 308 |
| $C_{3} \cup C_{8}$ | 81 |
| $C_{4} \cup C_{7}$ | 100 |
| $C_{5} \cup C_{6}$ | 70 |
| $2 C_{3} \cup C_{5}$ | 0 |
| $C_{3} \cup 2 C_{4}$ | 7 |
| $C_{13}$ | 3809 |
| $C_{3} \cup C_{10}$ | 578 |
| $C_{4} \cup C_{9}$ | 474 |
| $C_{5} \cup C_{8}$ | 495 |
| $C_{6} \cup C_{7}$ | 454 |
| $2 C_{3} \cup C_{7}$ | 103 |
| $C_{3} \cup C_{4} \cup C_{6}$ | 181 |
| $C_{3} \cup 2 C_{5}$ | 103 |
| $2 C_{4} \cup C_{5}$ | 97 |
| $3 C_{3} \cup C_{4}$ | 0 |
| $C_{15}$ | 63995 |
| $C_{3} \cup C_{12}$ | 11703 |
| $C_{4} \cup C_{11}$ | 11655 |
| $C_{5} \cup C_{10}$ | 9472 |
| $C_{6} \cup C_{9}$ | 9252 |
| $2 C_{3} \cup C_{9}$ | 1151 |
| $C_{7} \cup C_{8}$ | 8567 |
| $C_{3} \cup C_{4} \cup C_{8}$ | 2297 |
| $C_{3} \cup C_{5} \cup C_{7}$ | 1758 |
| $2 C_{4} \cup C_{7}$ | 1389 |
| $C_{3} \cup 2 C_{6}$ | 1117 |
| $C_{4} \cup C_{5} \cup C_{6}$ | 2023 |
| $3 C_{3} \cup C_{6}$ | 104 |
| $3 C_{5}$ | 328 |
| $2 C_{3} \cup C_{4} \cup C_{5}$ | 210 |
| $C_{3} \cup 3 C_{4}$ | 128 |
| $5 C_{3}$ | 11 |
|  |  |


| Graph | \#sVMTLs |
| ---: | ---: |
| $C_{17}$ | 1152784 |
| $C_{3} \cup C_{14}$ | 201685 |
| $C_{4} \cup C_{13}$ | 193899 |
| $C_{5} \cup C_{12}$ | 159485 |
| $C_{6} \cup C_{11}$ | 144516 |
| $2 C_{3} \cup C_{11}$ | 19625 |
| $C_{7} \cup C_{10}$ | 137561 |
| $C_{3} \cup C_{4} \cup C_{10}$ | 38453 |
| $C_{8} \cup C_{9}$ | 133174 |
| $C_{3} \cup C_{5} \cup C_{9}$ | 32242 |
| $2 C_{4} \cup C_{9}$ | 18545 |
| $C_{3} \cup C_{6} \cup C_{8}$ | 29515 |
| $C_{4} \cup C_{5} \cup C_{8}$ | 32697 |
| $3 C_{3} \cup C_{8}$ | 980 |
| $C_{3} \cup 2 C_{7}$ | 15499 |
| $C_{4} \cup C_{6} \cup C_{7}$ | 30576 |
| $2 C_{5} \cup C_{7}$ | 13651 |
| $2 C_{3} \cup C_{4} \cup C_{7}$ | 3792 |
| $C_{5} \cup 2 C_{6}$ | 13566 |
| $2 C_{3} \cup C_{5} \cup C_{6}$ | 3300 |
| $C_{3} \cup 2 C_{4} \cup C_{6}$ | 3429 |
| $C_{3} \cup C_{4} \cup 2 C_{5}$ | 3542 |
| $3 C_{4} \cup C_{5}$ | 1373 |
| $4 C_{3} \cup C_{5}$ | 94 |
| $3 C_{3} \cup 2 C_{4}$ | 174 |

Table 5: The number of strong VMTLs of 2-regular graphs of orders 19 and 21

| Graph | \#VMTLs |
| ---: | ---: |
| $C_{3} \cup C_{16}$ | 32319312 |
| $C_{4} \cup C_{15}$ | 5654282 |
| $C_{5} \cup C_{14}$ | 4339337 |
| $C_{6} \cup C_{13}$ | 3945295 |
| $2 C_{3} \cup C_{13}$ | 503413 |
| $C_{7} \cup C_{12}$ | 3638562 |
| $C_{3} \cup C_{4} \cup C_{12}$ | 985403 |
| $C_{8} \cup C_{11}$ | 3533924 |
| $C_{3} \cup C_{5} \cup C_{11}$ | 809230 |
| $2 C_{4} \cup C_{11}$ | 492587 |
| $C_{9} \cup C_{10}$ | 3404994 |
| $C_{3} \cup C_{6} \cup C_{10}$ | 744237 |
| $C_{4} \cup C_{5} \cup C_{10}$ | 788795 |
| $3 C_{3} \cup C_{10}$ | 31369 |
| $C_{3} \cup C_{7} \cup C_{9}$ | 705588 |
| $C_{4} \cup C_{6} \cup C_{9}$ | 740368 |
| $2 C_{5} \cup C_{9}$ | 331059 |
| $2 C_{3} \cup C_{4} \cup C_{9}$ | 96615 |
| $C_{3} \cup 2 C_{8}$ | 360721 |
| $C_{4} \cup C_{7} \cup C_{8}$ | 725134 |
| $C_{5} \cup C_{6} \cup C_{8}$ | 630100 |
| $2 C_{3} \cup C_{5} \cup C_{8}$ | 83476 |
| $C_{3} \cup 2 C_{4} \cup C_{8}$ | 102094 |
| $C_{5} \cup 2 C_{7}$ | 305346 |
| $2 C_{6} \cup C_{7}$ | 298900 |
| $2 C_{3} \cup C_{6} \cup C_{7}$ | 77161 |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{7}$ | 166608 |
| $3 C_{4} \cup C_{7}$ | 35318 |
| $4 C_{3} \cup C_{7}$ | 1559 |
| $C_{3} \cup C_{4} \cup 2 C_{6}$ | 82740 |
| $C_{3} \cup 2 C_{5} \cup C_{6}$ | 74490 |
| $2 C_{4} \cup C_{5} \cup C_{6}$ | 88441 |
| $3 C_{3} \cup C_{4} \cup C_{6}$ | 6432 |
| $C_{4} \cup 3 C_{5}$ | 25734 |
| $3 C_{3} \cup 2 C_{5}$ | 3187 |
| $2 C_{3} \cup 2 C_{4} \cup C_{5}$ | 11174 |
| $C_{3} \cup 4 C_{4}$ | 2389 |
| $5 C_{3} \cup C_{4}$ | 98 |


| Graph | \#sVMTLs |
| :---: | :---: |
| $C_{21}$ | $\geq 27377353$ |
| $C_{3} \cup C_{18}$ | $\geq 4936121$ |
| $C_{4} \cup C_{17}$ | $\geq 4402284$ |
| $C_{5} \cup C_{16}$ | $\geq 3560381$ |
| $C_{6} \cup C_{15}$ | $\geq 3196063$ |
| $2 C_{3} \cup C_{15}$ | $\geq 451408$ |
| $C_{7} \cup C_{14}$ | $\geq 2921701$ |
| $C_{3} \cup C_{4} \cup C_{14}$ | $\geq 831559$ |
| $C_{8} \cup C_{13}$ | $\geq 2755931$ |
| $C_{3} \cup C_{5} \cup C_{13}$ | $\geq 671181$ |
| $2 C_{4} \cup C_{13}$ | $\geq 376272$ |
| $C_{9} \cup C_{12}$ | $\geq 2673503$ |
| $C_{3} \cup C_{6} \cup C_{12}$ | $\geq 621505$ |
| $C_{4} \cup C_{5} \cup C_{12}$ | $\geq 623774$ |
| $3 C_{3} \cup C_{12}$ | $\geq 27971$ |
| $C_{10} \cup C_{11}$ | $\geq 2606600$ |
| $C_{3} \cup C_{7} \cup C_{11}$ | $\geq 578169$ |
| $C_{4} \cup C_{6} \cup C_{11}$ | $\geq 565461$ |
| $2 C_{5} \cup C_{11}$ | $\geq 255795$ |
| $2 C_{3} \cup C_{4} \cup C_{11}$ | $\geq 81011$ |
| $C_{3} \cup C_{8} \cup C_{10}$ | $\geq 549727$ |
| $C_{4} \cup C_{7} \cup C_{10}$ | $\geq 534062$ |
| $C_{5} \cup C_{6} \cup C_{10}$ | $\geq 476410$ |
| $2 C_{3} \cup C_{5} \cup C_{10}$ | $\geq 64867$ |
| $C_{3} \cup 2 C_{4} \cup C_{10}$ | $\geq 75250$ |
| $C_{3} \cup 2 C_{9}$ | $\geq 279738$ |
| $C_{4} \cup C_{8} \cup C_{9}$ | $\geq 519379$ |
| $C_{5} \cup C_{7} \cup C_{9}$ | $\geq 453412$ |
| $2 C_{6} \cup C_{9}$ | $\geq 223923$ |
| $2 C_{3} \cup C_{6} \cup C_{9}$ | $\geq 64974$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{9}$ | $\geq 129006$ |
| $3 C_{4} \cup C_{9}$ | $\geq 23778$ |
| $4 C_{3} \cup C_{9}$ | $\geq 1523$ |
| $C_{5} \cup 2 C_{8}$ | $\geq 223829$ |
| $C_{6} \cup C_{7} \cup C_{8}$ | $\geq 425393$ |
| $2 C_{3} \cup C_{7} \cup C_{8}$ | $\geq 59114$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup C_{8}$ | $\geq 120008$ |
| $C_{3} \cup 2 C_{5} \cup C_{8}$ | $\geq 53337$ |
| $2 C_{4} \cup C_{5} \cup C_{8}$ | $\geq 61719$ |
| $3 C_{3} \cup C_{4} \cup C_{8}$ | $\geq 5488$ |
| $3 C_{7}$ | $\geq 70429$ |
| $C_{3} \cup C_{4} \cup 2 C_{7}$ | $\geq 59441$ |
| $C_{3} \cup C_{5} \cup C_{6} \cup C_{7}$ | $\geq 104239$ |
| $2 C_{4} \cup C_{6} \cup C_{7}$ | $\geq 57074$ |
| $C_{4} \cup 2 C_{5} \cup C_{7}$ | $\geq 52371$ |
| $3 C_{3} \cup C_{5} \cup C_{7}$ | $\geq 4458$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{7}$ | $\geq 8365$ |
| $C_{3} \cup 3 C_{6}$ | $\geq 17476$ |
| $C_{4} \cup C_{5} \cup 2 C_{6}$ | $\geq 52295$ |
| $3 C_{3} \cup 2 C_{6}$ | $\geq 2697$ |
| $3 C_{5} \cup C_{6}$ | $\geq 15316$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{6}$ | $\geq 14546$ |
| $C_{3} \cup 3 C_{4} \cup C_{6}$ | $\geq 5218$ |
| $5 C_{3} \cup C_{6}$ | $\geq 70$ |
| $2 C_{3} \cup 3 C_{5}$ | $\geq 2075$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{5}$ | $\geq 7545$ |
| $4 C_{4} \cup C_{5}$ | $\geq 1418$ |
| $4 C_{3} \cup C_{4} \cup C_{5}$ | $\geq 318$ |
| $3 C_{3} \cup 3 C_{4}$ | $\geq 268$ |
| $7 C_{3}$ | $\geq 1$ |

Table 6: The number of strong VMTLs of 2-regular graphs of order 23

| Graph | \#VMTLs |
| ---: | ---: | ---: |
| $C_{3} \cup C_{23}$ | $\geq 26357261757$ |
| $C_{4} \cup C_{19}$ | $\geq 4448603705$ |
| $C_{5} \cup C_{18}$ | $\geq 330592148$ |
| $C_{6} \cup C_{17}$ | $\geq 2974916517$ |
| $2 C_{3} \cup C_{17}$ | $\geq 379559991$ |
| $C_{7} \cup C_{16}$ | $\geq 2704273294$ |
| $C_{3} \cup C_{4} \cup C_{16}$ | $\geq 742729581$ |
| $C_{8} \cup C_{15}$ | $\geq 2526728483$ |
| $C_{3} \cup C_{5} \cup C_{15}$ | $\geq 580163699$ |
| $2 C_{4} \cup C_{15}$ | $\geq 359145403$ |
| $C_{9} \cup C_{14}$ | $\geq 2407608798$ |
| $C_{3} \cup C_{6} \cup C_{14}$ | $\geq 530970815$ |
| $C_{4} \cup C_{5} \cup C_{14}$ | $\geq 565901123$ |
| $3 C_{3} \cup C_{14}$ | $\geq 21933380$ |
| $C_{10} \cup C_{13}$ | $\geq 2327937652$ |
| $C_{3} \cup C_{7} \cup C_{13}$ | $\geq 488030011$ |
| $C_{4} \cup C_{6} \cup C_{13}$ | $\geq 516909029$ |
| $2 C_{5} \cup C_{13}$ | $\geq 224320262$ |
| $2 C_{3} \cup C_{4} \cup C_{13}$ | $\geq 66525538$ |
| $C_{11} \cup C_{12}$ | $\geq 2297905959$ |
| $C_{3} \cup C_{8} \cup C_{12}$ | $\geq 464090043$ |
| $C_{4} \cup C_{7} \cup C_{12}$ | $\geq 479854273$ |
| $C_{5} \cup C_{6} \cup C_{12}$ | $\geq 414079520$ |
| $2 C_{3} \cup C_{5} \cup C_{12}$ | $\geq 52395655$ |
| $C_{3} \cup 2 C_{4} \cup C_{12}$ | $\geq 66327263$ |
| $C_{3} \cup C_{9} \cup C_{11}$ | $\geq 449781090$ |
| $C_{4} \cup C_{8} \cup C_{11}$ | $\geq 459482272$ |
| $C_{5} \cup C_{7} \cup C_{11}$ | $\geq 386223975$ |
| $2 C_{6} \cup C_{11}$ | $\geq 192003657$ |
| $2 C_{3} \cup C_{6} \cup C_{11}$ | $\geq 49095941$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{11}$ | $\geq 105760612$ |
| $3 C_{4} \cup C_{11}$ | $\geq 21929686$ |
| $4 C_{3} \cup C_{11}$ | $\geq 983805$ |
| $C_{3} \cup 2 C_{10}$ | $\geq 221878162$ |
| $C_{4} \cup C_{9} \cup C_{10}$ | $\geq 447724203$ |
| $C_{5} \cup C_{8} \cup C_{10}$ | $\geq 371268954$ |
| $C_{6} \cup C_{7} \cup C_{10}$ | $\geq 360490123$ |
| $2 C_{3} \cup C_{7} \cup C_{10}$ | $\geq 45951660$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup C_{10}$ | $\geq 98980991$ |
| $C_{3} \cup 2 C_{5} \cup C_{10}$ | $\geq 42422741$ |
| $2 C_{4} \cup C_{5} \cup C_{10}$ | $\geq 52778689$ |
| $3 C_{3} \cup C_{4} \cup C_{10}$ | $\geq 4131598$ |
| $C_{5} \cup 2 C_{9}$ | $\geq 184031336$ |


| $\mathrm{Graph}^{2}$ | \#sVMTLs |
| ---: | ---: | ---: |
| $C_{6} \cup C_{8} \cup C_{9}$ | $\geq 351538778$ |
| $2 C_{3} \cup C_{8} \cup C_{9}$ | $\geq 44865023$ |
| $2 C_{7} \cup C_{9}$ | $\geq 171643798$ |
| $C_{3} \cup C_{4} \cup C_{7} \cup C_{9}$ | $\geq 94074872$ |
| $C_{3} \cup C_{5} \cup C_{6} \cup C_{9}$ | $\geq 80858145$ |
| $2 C_{4} \cup C_{6} \cup C_{9}$ | $\geq 49741395$ |
| $C_{4} \cup 2 C_{5} \cup C_{9}$ | $\geq 43259847$ |
| $3 C_{3} \cup C_{5} \cup C_{9}$ | $\geq 3313463$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{9}$ | $\geq 6444557$ |
| $C_{7} \cup 2 C_{8}$ | $\geq 169185964$ |
| $C_{3} \cup C_{4} \cup 2 C_{8}$ | $\geq 46681533$ |
| $C_{3} \cup C_{5} \cup C_{7} \cup C_{8}$ | $\geq 77600357$ |
| $2 C_{4} \cup C_{7} \cup C_{8}$ | $\geq 48191402$ |
| $C_{3} \cup 2 C_{6} \cup C_{8}$ | $\geq 38849530$ |
| $C_{4} \cup C_{5} \cup C_{6} \cup C_{8}$ | $\geq 82607864$ |
| $3 C_{3} \cup C_{6} \cup C_{8}$ | $\geq 3223654$ |
| $3 C_{5} \cup C_{8}$ | $\geq 11883659$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{8}$ | $\geq 10597864$ |
| $C_{3} \cup 3 C_{4} \cup C_{8}$ | $\geq 4493631$ |
| $5 C_{3} \cup C_{8}$ | $\geq 37176$ |
| $C_{3} \cup C_{6} \cup 2 C_{7}$ | $\geq 37790794$ |
| $C_{4} \cup C_{5} \cup 2 C_{7}$ | $\geq 40337251$ |
| $3 C_{3} \cup 2 C_{7}$ | $\geq 1560680$ |
| $C_{4} \cup 2 C_{6} \cup C_{7}$ | $\geq 39980575$ |
| $2 C_{5} \cup C_{6} \cup C_{7}$ | $\geq 34768100$ |
| $2 C_{3} \cup C_{4} \cup C_{6} \cup C_{7}$ | $\geq 10283687$ |
| $2 C_{3} \cup 2 C_{5} \cup C_{7}$ | $\geq 4358242$ |
| $C_{3} \cup 2 C_{4} \cup C_{5} \cup C_{7}$ | $\geq 11103433$ |
| $4 C_{4} \cup C_{7}$ | $\geq 1163951$ |
| $4 C_{3} \cup C_{4} \cup C_{7}$ | $\geq 208467$ |
| $C_{5} \cup 3 C_{6}$ | $\geq 11565138$ |
| $2 C_{3} \cup C_{5} \cup 2 C_{6}$ | $\geq 4412305$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{6}$ | $\geq 5506269$ |
| $C_{3} \cup C_{4} \cup 2 C_{5} \cup C_{6}$ | $\geq 9504325$ |
| $3 C_{4} \cup C_{5} \cup C_{6}$ | $\geq 3915430$ |
| $4 C_{3} \cup C_{5} \cup C_{6}$ | $\geq 174271$ |
| $3 C_{3} \cup 2 C_{4} \cup C_{6}$ | $\geq 465251$ |
| $C_{3} \cup 4 C_{5}$ | $\geq 670652$ |
| $2 C_{4} \cup 3 C_{5}$ | $\geq 1699135$ |
| $3 C_{3} \cup C_{4} \cup 2 C_{5}$ | $\geq 391763$ |
| $2 C_{3} \cup 3 C_{4} \cup C_{5}$ | $\geq 509010$ |
| $6 C_{3} \cup C_{5}$ | $\geq 1191$ |
| $C_{3} \cup 5 C_{4}$ | $\geq 66757$ |
| $5 C_{3} \cup 2 C_{4}$ | $\geq 5434$ |

There is much stronger evidence for Conjecture 1, however. The data presented in this paper pertains to strong labelings, where the magic constant has the smallest feasible value. The very earliest studies of VMTLs of graphs led us to realize that values of the magic constant near the middle of the range of feasible values almost always allowed many more VMTLs than values near the extremes of the range, usually by several orders of magnitude. With a few small exceptions, the magic constant spectrum of a regular graph produces a classical bell-shaped distribution of counts (the symmetry is due to the duality of labelings for regular graphs [5]). We illustrate this with an example. The 3 -cube $Q_{3}$ is a 3 -regular graph of order 8 for which the feasible magic constant $k$ lies in the range $36 \leq k \leq 48$. We counted the number of VMTLs for each value of $k$ and the results are shown in Table 7 below.

Table 7: Distribution of number of VMTLs for $Q_{3}$

| $k$ | $N$ |
| ---: | ---: |
| 36 | 3048 |
| 37 | 6997 |
| 38 | 30788 |
| 39 | 50765 |
| 40 | 117101 |
| 41 | 132358 |
| 42 | 258575 |
| 43 | 132358 |
| 44 | 117101 |
| 45 | 50765 |
| 46 | 30788 |
| 47 | 6997 |
| 48 | 3048 |
| $\Sigma$ | 940689 |

Finally we wish to remind the reader how powerful Gray's Theorem is. Using it we have now proved that all regular graphs of odd order less than 30 have strong VMTLs. How many graphs are we talking about? Table 8 tabulates the number of $r$-regular graphs of odd order $n$ as far as is known at present. For a fixed $r$ the number grows exponentially with $n$, of course, and the numbers become enormous quickly. The italic entries in the table are new values we have calculated that were not known previously. These were calculated using Markus Meringer's algorithm [8] for fast generation of regular graphs. The numbers in the table are recorded as sequences $\mathrm{A} 008483(n), \operatorname{A033301}(n)$ and $\mathrm{A} 165627(n)$ in the OEIS.

Table 8: The number of $r$-regular graphs of order $n$.

| $n$ | $r=2$ | $r=4$ | $r=6$ | $r=8$ | $r=10$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 3 | 1 |  |  |  |  |
| 5 | 1 | 1 |  |  |  |
| 7 | 2 | 2 | 1 |  |  |
| 9 | 4 | 16 | 4 | 1 |  |
| 11 | 6 | 266 | 266 | 6 | 1 |
| 13 | 10 | 10786 | 367860 | 10786 | 10 |
| 15 | 17 | 805579 | 1470293676 | 1470293676 | 805579 |
| 17 | 25 | 8623360 | 9799685588961 | $? ?$ | 9799685588961 |
| 19 | 39 | 11946592242 |  | $? ?$ |  |
| 21 | 60 | 2056701139136 |  |  | $? ?$ |
| 23 | 88 | $? ?$ |  |  |  |

The calculations for this paper were done on a cluster of standard Windows PCs in student labs and written using the Magma package [1]. The VMTLs were found using branch-and-bound search methods which focussed on the size of the longest cycle in the graph. Several variations of the search procedure were used, depending on the cycle-lengths involved. Finally we remark that searching for a strong VMTL was easier than searching for an arbitrary one because there was only one suitable set of vertex labels, and hence also only one suitable set of edge labels.

Table 9: Strong VMTLs of the 25 2-regular graphs of order 17

| Graph | Edge Labels |
| ---: | :--- |
| $C_{17}$ | $(1,13,2,14,10,11,9,17,8,15,3,16,6,4,7,5,12)$ |
| $C_{3} \cup C_{14}$ | $(1,15,2,9,5,14,12,3,7,6,16,4,8,17)(10,11,13)$ |
| $C_{4} \cup C_{13}$ | $(1,10,8,5,17,4,11,13,6,14,12,2,15)(3,9,16,7)$ |
| $C_{5} \cup C_{12}$ | $(1,15,2,12,11,14,7,17,5,13,6,9)(3,10,16,4,8)$ |
| $C_{6} \cup C_{11}$ | $(1,14,12,10,15,2,11,13,5,7,9)(3,16,4,17,6,8)$ |
| $2 C_{3} \cup C_{11}$ | $(1,14,10)(2,8,6,17,4,16,3,9,7,15,11)(5,13,12)$ |
| $C_{7} \cup C_{10}$ | $(1,9,8,15,11,14,6,16,5,13)(2,17,7,4,12,3,10)$ |
| $C_{3} \cup C_{4} \cup C_{10}$ | $(1,13,7,17,2,10,3,15,8,9)(4,11,14,12)(5,6,16)$ |
| $C_{8} \cup C_{9}$ | $(1,15,11,3,7,13,2,16)(4,8,17,6,5,14,10,12,9)$ |
| $C_{3} \cup C_{5} \cup C_{9}$ | $(1,15,9,14,12,7,13,2,16)(3,10,11)(4,8,17,5,6)$ |
| $2 C_{4} \cup C_{9}$ | $(1,16,2,13,6,8,17,5,15)(3,9,4,7)(10,14,12,11)$ |
| $C_{3} \cup C_{6} \cup C_{8}$ | $(1,9,11,4,8,13)(2,14,12,5,6,16,7,17)(3,10,15)$ |
| $C_{4} \cup C_{5} \cup C_{8}$ | $(1,13,4,11,9)(2,14,12,6,16,5,7,17)(3,10,15,8)$ |
| $3 C_{3} \cup C_{8}$ | $(1,13,2,17,6,5,16,9)(3,14,10)(4,8,12)(7,15,11)$ |
| $C_{3} \cup 2 C_{7}$ | $(1,13,6,17,5,10,16)(2,11,14,7,3,15,9)(4,8,12)$ |
| $C_{4} \cup C_{6} \cup C_{7}$ | $(1,13,2,8,3,10,16)(4,15,9,12)(5,7,11,14,6,17)$ |
| $2 C_{5} \cup C_{7}$ | $(1,13,2,10,16)(3,8,11,14,7)(4,12,6,17,5,15,9)$ |
| $2 C_{3} \cup C_{4} \cup C_{7}$ | $(1,16,8,13)(2,10,15,11,9,4,14)(3,12,7)(5,6,17)$ |
| $C_{5} \cup 2 C_{6}$ | $(1,11,8,7,13,12)(2,14,3,15,9)(4,10,16,5,17,6)$ |
| $2 C_{3} \cup C_{5} \cup C_{6}$ | $(1,12,7,14,11)(2,13,3,15,9,8)(4,10,16)(5,6,17)$ |
| $C_{3} \cup 2 C_{4} \cup C_{6}$ | $(1,11,14,12)(2,16,8,9)(3,13,7)(4,15,6,17,5,10)$ |
| $C_{3} \cup C_{4} \cup 2 C_{5}$ | $(1,15,9,2,11)(3,17,8,7)(4,13,10)(5,14,12,6,16)$ |
| $3 C_{4} \cup C_{5}$ | $(1,15,2,11)(3,16,8,17)(4,7,14,12,6)(5,9,13,10)$ |
| $4 C_{3} \cup C_{5}$ | $(1,15,11)(2,13,12)(3,14,7)(4,9,10,8,16)(5,6,17)$ |
| $3 C_{3} \cup 2 C_{4}$ | $(1,17,4,12)(2,13,10)(3,7,15,11)(5,14,6)(8,9,16)$ |

Table 10: Strong VMTLs of the 39 2-regular graphs of order 19
$\left.\begin{array}{|r|l|}\hline \text { Graph } & \text { Edge Labels } \\ \hline C_{19} & (1,12,2,17,7,18,3,9,6,5,11,15,13,14,8,10,19,4,16) \\ C_{3} \cup C_{16} & (1,16,10,19,9,4,17,2,13,11,3,8,14,6,12,15)(5,7,18) \\ C_{4} \cup C_{15} & (1,11,6,17,3,19,7,18,10,4,15,12,9,2,14)(5,8,16,13) \\ C_{5} \cup C_{14} & (1,16,7,4,11)(2,12,13,9,17,3,15,6,18,10,19,8,5,14) \\ C_{6} \cup C_{13} & (1,10,6,18,4,19)(2,15,11,17,12,13,14,7,8,5,9,3,16) \\ 2 C_{3} \cup C_{13} & (1,19,4,18,6,15,11,17,12,2,16,9,10)(3,14,13)(5,8,7) \\ C_{7} \cup C_{12} & (1,13,15,14,4,17,2,9,7,10,16,11)(3,12,8,5,18,6,19) \\ C_{3} \cup C_{4} \cup C_{12} & (1,13,3,19,6,18,5,15,14,12,16,11)(2,9,4,17)(7,10,8) \\ C_{8} \cup C_{11} & (1,10,19,2,16,11,4,8,18,7,12)(3,13,15,9,5,17,6,14) \\ C_{3} \cup C_{5} \cup C_{11} & (1,10,19,2,16,6,14,3,13,15,12)(4,11,8)(5,9,17,7,18) \\ 2 C_{4} \cup C_{11} & (1,10,19,2,16,4,15,11,17,5,12)(3,9,14,13)(6,8,7,18) \\ C_{9} \cup C_{10} & (1,16,13,2,10,17,3,15,6,18)(4,12,11,14,8,5,9,19,7) \\ C_{3} \cup C_{6} \cup C_{10} & (1,16,11,2,10,8,15,14,6,18)(3,12,13)(4,17,5,9,19,7) \\ C_{4} \cup C_{5} \cup C_{10} & (1,18,6,16)(2,13,5,8,15,14,11,3,17,10)(4,12,9,19,7) \\ 3 C_{3} \cup C_{10} & (1,18,2,10,5,9,19,7,6,16)(3,8,15)(4,17,12)(11,14,13) \\ C_{3} \cup C_{7} \cup C_{9} & (1,19,6,5,11,2,13)(3,15,8,4,18,9,17,12,16)(7,10,14) \\ C_{4} \cup C_{6} \cup C_{9} & (1,13,15,3,16,10,5,6,19)(2,14,7,17,12,11)(4,8,9,18) \\ 2 C_{5} \cup C_{9} & (1,19,6,15,13)(2,10,5,14,3,8,16,7,11)(4,18,9,17,12) \\ 2 C_{3} \cup C_{4} \cup C_{9} & (1,13,10,16,8,7,5,6,19)(2,11,17)(3,15,14)(4,12,9,18) \\ C_{3} \cup 2 C_{8} & (1,12,16,13,6,15,10,17)(2,18,4,11,3,8,9,14)(5,19,7) \\ C_{4} \cup C_{7} \cup C_{8} & (1,12,2,18,4,8,17)(3,13,15,14)(5,19,7,16,11,10,9,6) \\ C_{5} \cup C_{6} \cup C_{8} & (1,17,10,5,19,7,16,12)(2,9,8,4,18)(3,13,6,15,14,11) \\ 2 C_{3} \cup C_{5} \cup C_{8} & (1,17,2,18,4,8,3,12)(5,19,7,16,9)(6,11,10)(13,15,14) \\ C_{3} \cup 2 C_{4} \cup C_{8} & (1,12,11,17)(2,18,4,10,6,15,14,13)(3,8,9,16)(5,19,7) \\ C_{5} \cup 2 C_{7} & (1,15,9,2,13)(3,17,4,8,18,5,14)(6,7,11,16,12,10,19) \\ 2 C_{6} \cup C_{7} & (1,15,4,9,2,13)(3,14,8,18,5,7,17)(6,12,16,11,10,19) \\ 2 C_{3} \cup C_{6} \cup C_{7} & (1,15,9,3,17,2,13)(4,14,7)(5,8,18)(6,11,16,12,10,19) \\ C_{3} \cup C_{4} \cup C_{5} \cup C_{7} & (1,15,13)(2,16,11,9)(3,14,10,19,6,7,12)(4,8,18,5,17) \\ 3 C_{4} \cup C_{7} & (1,15,2,13)(3,8,18,5,17,4,9)(6,14,10,19)(7,12,16,11) \\ 4 C_{3} \cup C_{7} & (1,15,13)(2,16,10,17,4,18,11)(3,9,14)(5,19,6)(7,12,8) \\ 5 C_{3} \cup C_{4} & (1,10,15)(2,11,16)(3,17,12)(4,13,9,19)(5,14,7)(6,18,8) \\ C_{3} \cup C_{4} \cup 2 C_{6} & (1,17,11,14)(2,10,12,8,6,15)(3,16,13)(4,9,18,5,19,7) \\ C_{3} \cup 2 C_{5} \cup C_{6} & (1,17,4,12,11,14)(2,15,9,18,10)(3,16,13,7,19)(5,8,6) \\ 2 C_{4} \cup C_{5} \cup C_{6} & (1,17,8,6,14)(2,15,12,10)(3,16,5,18,11,13)(4,9,19,7) \\ 3 C_{3} \cup C_{4} \cup C_{6} & (1,17,8,14)(2,10,19,7,13,15)(3,16,11)(4,12,9)(5,18,6) \\ \hline & (1,11,12,2,14)(3,10,19,6,16)(4,7,17)(5,15,13)(8,9,18) \\ \hline & (11,14)(2,15,12)(3,13,8,16)(5,6,17)(7,18,10,19) \\ \hline\end{array}\right)$

Table 11: Strong VMTLs of the 60 2-regular graphs of order 21

| Graph | Edge Labels |
| :---: | :---: |
| $C_{21}$ | $(1,18,14,13,3,15,7,17,4,11,9,5,8,21,2,10,20,6,19,12,16)$ |
| $C_{3} \cup C_{18}$ | $(1,13,17,11,8,4,12,3,21,10,19,7,6,16,2,15,5,20)(9,18,14)$ |
| $C_{4} \cup C_{17}$ | $(1,16,9,15,6,14,4,11,5,7,20,3,19,13,17,2,12)(8,18,10,21)$ |
| $C_{5} \cup C_{16}$ | $(1,18,5,19,8,21,7,11,3,9,17,4,13,2,20,12)(6,14,16,15,10)$ |
| $C_{6} \cup C_{15}$ | $(1,11,20,12,4,14,10,15,2,13,16,6,17,3,18)(5,8,19,7,21,9)$ |
| $2 C_{3} \cup C_{15}$ | $(1,18,3,17,15,2,13,14,12,6,16,8,5,20,11)(4,10,19)(7,21,9)$ |
| $C_{7} \cup C_{14}$ | $(1,18,2,14,16,10,5,12,19,13,8,6,17,11)(3,21,4,9,20,7,15)$ |
| $C_{3} \cup C_{4} \cup C_{14}$ | $(1,18,2,14,17,15,3,21,4,9,13,10,19,11)(5,16,12)(6,20,7,8)$ |
| $C_{8} \cup C_{13}$ | $(1,12,11,15,16,4,13,5,19,10,20,7,21)(2,17,8,6,9,3,18,14)$ |
| $C_{3} \cup C_{5} \cup C_{13}$ | $(1,21,7,20,10,19,13,4,8,16,9,6,12)(2,17,14)(3,11,15,5,18)$ |
| $2 C_{4} \cup C_{13}$ | $(1,12,5,19,13,8,18,11,4,10,20,7,21)(2,17,6,14)(3,9,16,15)$ |
| $C_{9} \cup C_{12}$ | $(1,18,14,4,17,13,9,20,11)(2,12,16,7,8,19,6,10,3,21,5,15)$ |
| $C_{3} \cup C_{6} \cup C_{12}$ | $(1,11,20,9,6,19,8,14,4,12,2,18)(3,21,5,16,7,10)(13,17,15)$ |
| $C_{4} \cup C_{5} \cup C_{12}$ | $(1,11,20,9,6,19,8,10,3,21,5,18)(2,12,16,14)(4,17,15,7,13)$ |
| $3 C_{3} \cup C_{12}$ | $(1,18,14,2,10,5,21,3,15,7,16,12)(4,13,17)(6,8,19)(9,20,11)$ |
| $C_{10} \cup C_{11}$ | $(1,21,8,20,3,9,5,14,13,17,15)(2,19,12,6,18,7,10,16,4,11)$ |
| $C_{3} \cup C_{7} \cup C_{11}$ | $(1,21,8,20,3,17,15)(2,11,14,16,10,4,13,5,7,12,19)(6,9,18)$ |
| $C_{4} \cup C_{6} \cup C_{11}$ | $(1,21,8,20,3,10,7,5,9,17,15)(2,13,6,18,12,19)(4,14,11,16)$ |
| $2 C_{5} \cup C_{11}$ | $(1,21,8,20,3,11,16,10,7,5,15)(2,17,13,12,19)(4,9,6,18,14)$ |
| $2 C_{3} \cup C_{4} \cup C_{11}$ | $(1,21,8,20,3,10,16,4,13,17,15)(2,12,19)(5,14,11,7)(6,9,18)$ |
| $C_{3} \cup C_{8} \cup C_{10}$ | $(1,21,10,8,7,19,6,15,2,12)(3,13,17,11,16,4,20,9)(5,18,14)$ |
| $C_{4} \cup C_{7} \cup C_{10}$ | $(1,21,10,5,16,2,12)(3,13,6,14)(4,20,9,18,7,19,11,17,15,8)$ |
| $C_{5} \cup C_{6} \cup C_{10}$ | $(1,12,2,10,21)(3,13,5,15,8,19,7,18,14,16)(4,20,9,6,11,17)$ |
| $2 C_{3} \cup C_{5} \cup C_{10}$ | $(1,12,2,10,21)(3,18,14,16,9,20,4,15,5,13)(6,11,17)(7,8,19)$ |
| $C_{3} \cup 2 C_{4} \cup C_{10}$ | $(1,21,10,18,14,13,3,16,2,12)(4,20,9,17)(5,15,8,7)(6,19,11)$ |
| $C_{3} \cup 2 C_{9}$ | $(1,20,5,14,2,11,6,18,13)(3,9,21,7,8,10,16,4,19)(12,17,15)$ |
| $C_{4} \cup C_{8} \cup C_{9}$ | $(1,13,4,15,16,10,5,20)(2,11,18,14)(3,19,8,12,6,17,7,21,9)$ |
| $C_{5} \cup C_{7} \cup C_{9}$ | $(1,20,5,18,6,11,2,14,13)(3,19,12,17,15)(4,8,7,21,9,10,16)$ |
| $2 C_{6} \cup C_{9}$ | $(1,13,16,10,5,20)(2,11,6,17,7,21,9,18,14)(3,15,4,8,12,19)$ |
| $2 C_{3} \cup C_{6} \cup C_{9}$ | $(1,13,2,11,5,20)(3,19,8,18,14,10,7,21,9)(4,15,16)(6,17,12)$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{9}$ | $(1,13,5,20)(2,11,18,6,14)(3,19,12)(4,8,9,21,7,16,10,17,15)$ |
| $3 C_{4} \cup C_{9}$ | $(1,20,5,13)(2,18,14,17,7,21,9,6,11)(3,19,10,16)(4,8,15,12)$ |
| $4 C_{3} \cup C_{9}$ | $(1,20,5,19,8,4,18,2,13)(3,10,16)(6,12,11)(7,21,9)(14,17,15)$ |
| $C_{5} \cup 2 C_{8}$ | $(1,20,7,16,6,14,2,12)(3,9,10,5,19,11,17,15)(4,21,8,18,13)$ |
| $C_{6} \cup C_{7} \cup C_{8}$ | $(1,12,2,14,16,7,20)(3,9,17,15,5,19)(4,11,6,13,18,10,8,21)$ |
| $2 C_{3} \cup C_{7} \cup C_{8}$ | $(1,20,7,11,3,19,5,12)(2,18,14)(4,21,8)(6,17,13,15,16,10,9)$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup C_{8}$ | $(1,12,7,20)(2,15,16,10,18,14)(3,19,5,13,17,6,9,11)(4,21,8)$ |
| $C_{3} \cup 2 C_{5} \cup C_{8}$ | $(1,20,7,16,10,9,6,12)(2,15,13,18,14)(3,11,19,5,17)(4,21,8)$ |
| $2 C_{4} \cup C_{5} \cup C_{8}$ | $(1,12,5,19,3,11,7,20)(2,10,18,14)(4,21,8,15,16)(6,13,17,9)$ |
| $3 C_{3} \cup C_{4} \cup C_{8}$ | $(1,20,7,12)(2,16,14)(3,11,6,9,17,15,5,19)(4,21,8)(10,18,13)$ |
| $3 C_{7}$ | $(1,11,12,19,13,7,15)(2,16,14,3,18,10,17)(4,20,9,6,8,5,21)$ |
| $C_{3} \cup C_{4} \cup 2 C_{7}$ | $(1,11,20,4,21,5,15)(2,16,12,10,19,13,17)(3,14,9,18)(6,8,7)$ |
| $C_{3} \cup C_{5} \cup C_{6} \cup C_{7}$ | $(1,11,6,7,13,8,15)(2,17,10,18,14,16)(3,19,12)(4,21,5,9,20)$ |
| $2 C_{4} \cup C_{6} \cup C_{7}$ | $(1,11,12,15)(2,16,14,17)(3,19,13,7,10,18)(4,20,9,6,8,5,21)$ |
| $C_{4} \cup 2 C_{5} \cup C_{7}$ | $(1,11,6,7,8,12,15)(2,16,14,17)(3,19,13,10,18)(4,21,5,9,20)$ |
| $3 C_{3} \cup C_{5} \cup C_{7}$ | $(1,11,20,4,21,5,15)(2,12,16)(3,10,19)(6,17,13)(7,8,9,18,14)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{7}$ | $(1,11,17,15)(2,16,3,18)(4,21,5,12,19,10,20)(6,8,7)(9,14,13)$ |
| $C_{3} \cup 3 C_{6}$ | $(1,20,8,5,7,16)(2,17,3,15,12,14)(4,10,21)(6,18,11,19,13,9)$ |
| $C_{4} \cup C_{5} \cup 2 C_{6}$ | $(1,16,7,5,8,20)(2,17,9,6,18)(3,11,19,13,14,15)(4,12,10,21)$ |
| $3 C_{3} \cup 2 C_{6}$ | $(1,16,3,19,8,20)(2,13,5,7,6,14)(4,10,21)(9,17,15)(11,18,12)$ |
| $3 C_{5} \cup C_{6}$ | $(1,20,8,6,7,16)(2,17,15,5,13)(3,19,11,18,9)(4,12,14,10,21)$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{6}$ | $(1,16,6,7,8,20)(2,17,12,14)(3,9,18,5,15)(4,10,21)(11,19,13)$ |
| $C_{3} \cup 3 C_{4} \cup C_{6}$ | $(1,16,11,15,8,20)(2,17,12,18)(3,19,13)(4,14,10,21)(5,9,6,7)$ |
| $5 C_{3} \cup C_{6}$ | $(1,20,9,3,12,16)(2,18,14)(4,10,21)(5,8,19)(6,13,17)(7,11,15)$ |
| $2 C_{3} \cup 3 C_{5}$ | $(1,16,5,15,13)(2,21,8,10,17)(3,9,7,6,19)(4,20,11)(12,18,14)$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{5}$ | $(1,16,14,13)(2,17,15,5,21)(3,19,12)(4,9,20,8)(6,18,7,11,10)$ |
| $4 C_{4} \cup C_{5}$ | $(1,13,5,16)(2,21,7,17)(3,19,11,20,12)(4,9,18,8)(6,14,15,10)$ |
| $4 C_{3} \cup C_{4} \cup C_{5}$ | $(1,13,16)(2,21,11,17)(3,9,15)(4,18,8,19,12)(5,10,20)(6,14,7)$ |
| $3 C_{3} \cup 3 C_{4}$ | $(1,15,8,20)(2,13,17)(3,11,18,14)(4,9,16)(5,19,7)(6,12,10,21)$ |
| $7 C_{3}$ | $(1,18,14)(2,15,16)(3,13,17)(4,21,8)(5,19,9)(6,20,7)(10,11,12)$ |

Table 12: Strong VMTLs of the first 50 2-regular graphs of order 23

| Graph | Edge Labels |
| :---: | :---: |
| $C_{23}$ | ( $1,17,2,12,19,4,20,15,5,21,7,18,9,8,13,3,10,22,11,23,6,16,14)$ |
| $C_{3} \cup C_{20}$ | (1,16, 13, 7, 9, 23, 3, 15, 19, 4, 18, 6, 22, 5, 8, 17, 2, 12, 21, 14)(10, 11, 20) |
| $C_{4} \cup C_{19}$ | $(1,23,6,9,5,15,17,10,20,8,18,3,22,11,2,14,4,13,21)(7,12,19,16)$ |
| $C_{5} \cup C_{18}$ | $(1,13,2,16,15,9,8,19,7,23,6,22,11,21,14,20,5,18)(3,17,4,12,10)$ |
| $C_{6} \cup C_{17}$ | $(1,21,9,23,5,19,16,18,11,20,13,7,14,2,15,4,22)(3,12,6,8,17,10)$ |
| $2 C_{3} \cup C_{17}$ | (1,21,9, 23, 5, 14, 2, 15, 19, 16, 11, 20, 13, 8, 10, 4, 22)(3, 12, 17) (6, 7, 18) |
|  | $(1,15,3,14,10,20,11,8,7,13,19,2,12,23,6,22)(4,21,5,17,16,18,9)$ |
| $C_{3} \cup C_{4} \cup C_{16}$ | $(1,15,16,18,14,4,21,5,10,3,19,2,12,23,6,22)(7,17,13,20)(8,9,11)$ |
| $C_{8} \cup C_{15}$ | (1,17, 16, 7, 6, 23, 8, 19)(2, 20, 15, 9, 21, 4, 22, 12, 5, 14, 18, 10, 11, 3, 13) |
| $C_{3} \cup C_{5} \cup C_{15}$ | $(1,17,11,16,7,10,3,13,2,20,15,18,14,5,19)(4,22,12,9,21)(6,23,8)$ |
| $2 C_{4} \cup C_{15}$ | (1, 19, 8, 23, 6, 11, 5, 9, 21, 4, 22, 12, 7, 16, 17) (2, 20, 15, 13) (3, 10, 14, 18) |
| $C_{9} \cup C_{14}$ | $(1,12,2,20,15,17,14,9,7,8,10,23,4,16)(3,22,6,13,21,5,19,11,18)$ |
| $C_{3} \cup C_{6} \cup C_{14}$ | (1, 12, 2, 20, 15, 9, 7, 8, 11, 19, 10, 23, 4, 16)(3, 22, 6, 17, 14, 18)(5, 13, 21) |
| $C_{4} \cup C_{5} \cup C_{14}$ | $(1,12,2,20,15,19,13,17,14,9,10,23,4,16)(3,22,6,18)(5,11,7,8,21)$ |
| $3 C_{3} \cup C_{14}$ | $(1,16,4,23,10,5,19,15,3,22,6,20,2,12)(7,9,14)(8,11,21)(13,18,17)$ |
| $C_{10} \cup C_{13}$ | $(1,20,15,2,16,9,21,13,18,14,8,7,12)(3,11,5,19,10,23,4,22,6,17)$ |
| $C_{3} \cup C_{7} \cup C_{13}$ | (1,20, 15, 17, 7, 8, 6, 22, 4, 23, 10, 19, 12)(2, 16, 3, 14, 11, 5, 18) (9, 21, 13) |
| $C_{4} \cup C_{6} \cup C_{13}$ | (1,20, 15, 2, 16, 3, 11, 13, 21, 9, 7, 8, 12)(4,22, 6, 19, 10, 23)(5, 17, 14, 18) |
| $2 C_{5} \cup C_{13}$ | $(1,20,15,2,16,3,17,14,8,7,18,11,12)(4,22,6,10,23)(5,19,13,21,9)$ |
| $2 C_{3} \cup C_{4} \cup C_{13}$ | $(1,20,15,16,3,19,10,23,4,22,6,11,12)(2,18,14)(5,13,21,9)(7,17,8)$ |
| $C_{11} \cup C_{12}$ | $(1,22,7,12,23,3,13,21,6,16,2,19)(4,10,20,5,8,9,15,18,14,17,11)$ |
| $C_{3} \cup C_{8} \cup C_{12}$ | (1, 19, 2, 16, 14, 17, 15, 10, 9, 5, 11, 22)(3, 12, 23) (4, 18, 6, 7, 20, 8, 21, 13) |
| $C_{4} \cup C_{7} \cup C_{12}$ | $(1,22,12,23,3,13,15,18,6,16,2,19)(4,11,21,9,20,7,10)(5,8,17,14)$ |
| $C_{5} \cup C_{6} \cup C_{12}$ | $(1,22,6,16,2,19)(3,13,21,9,8,5,14,17,7,20,12,23)(4,10,15,18,11)$ |
| $2 C_{3} \cup C_{5} \cup C_{12}$ | $(1,22,6,10,15,14,20,7,17,16,2,19)(3,12,23)(4,13,18)(5,8,11,21,9)$ |
| $C_{3} \cup 2 C_{4} \cup C_{12}$ | $(1,19,2,16,9,5,14,20,11,21,8,22)(3,12,23)(4,18,15,13)(6,7,17,10)$ |
| $C_{3} \cup C_{9} \cup C_{11}$ | $(1,17,12,23,3,20,11,4,16)(2,22,10,6,8,5,14,13,21,9,19)(7,18,15)$ |
| $C_{4} \cup C_{8} \cup C_{11}$ | $(1,16,4,11,5,9,18,15,14,8,17)(2,19,12,23,3,20,10,22)(6,13,21,7)$ |
| $C_{5} \cup C_{7} \cup C_{11}$ | $(1,16,4,11,17)(2,22,10,6,21,9,5,8,14,15,19)(3,20,13,18,7,12,23)$ |
| $2 C_{6} \cup C_{11}$ | $(1,17,14,15,4,16)(2,22,10,5,11,19)(3,20,8,6,7,18,9,13,21,12,23)$ |
| $2 C_{3} \cup C_{6} \cup C_{11}$ | $(1,16,4,18,13,15,19,2,22,10,17)(3,20,9,21,12,23)(5,14,11)(6,8,7)$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{11}$ | $(1,16,4,11,17)(2,19,12,23,3,20,14,13,6,10,22)(5,8,21,9)(7,18,15)$ |
| $3 C_{4} \cup C_{11}$ | $(1,17,14,16)(2,22,10,19)(3,20,8,6,7,18,9,13,21,12,23)(4,11,5,15)$ |
| $4 C_{3} \cup C_{11}$ | $(1,17,16)(2,22,10,20,11,5,14,15,8,6,19)(3,12,23)(4,18,9)(7,21,13)$ |
| $C_{3} \cup 2 C_{10}$ | $(1,23,10,4,12,5,15,19,11,17)(2,13,22,9,16,3,18,14,8,21)(6,7,20)$ |
| $C_{4} \cup C_{9} \cup C_{10}$ | $(1,23,10,20,7,19,6,15,17)(2,13,22,9,11,3,14,5,8,21)(4,18,16,12)$ |
| $C_{5} \cup C_{8} \cup C_{10}$ | $(1,23,10,15,17)(2,13,22,9,19,11,3,18,8,21)(4,12,5,14,20,7,6,16)$ |
| $C_{6} \cup C_{7} \cup C_{10}$ | $(1,23,10,18,3,14,11,19,15,17)(2,13,22,9,5,8,21)(4,12,7,20,6,16)$ |
| $2 C_{3} \cup C_{7} \cup C_{10}$ | $(1,23,10,4,12,14,8,5,15,17)(2,13,22,9,20,7,21)(3,18,16)(6,11,19)$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup C_{10}$ | $(1,23,10,20,5,17)(2,13,22,9,18,14,3,11,8,21)(4,12,16)(6,15,19,7)$ |
| $C_{3} \cup 2 C_{5} \cup C_{10}$ | $(1,23,10,3,17)(2,13,22,9,19,15,11,6,8,21)(4,12,20,7,18)(5,16,14)$ |
| $2 C_{4} \cup C_{5} \cup C_{10}$ | $(1,23,10,4,17)(2,13,22,9,19,15,11,5,8,21)(3,14,6,16)(7,18,12,20)$ |
| $3 C_{3} \cup C_{4} \cup C_{10}$ | $(1,23,10,17)(2,15,4,16,13,22,9,5,11,21)(3,12,18)(6,7,19)(8,20,14)$ |
| $C_{5} \cup 2 C_{9}$ | $(1,19,8,15,3,23,11,2,20)(4,10,5,12,21)(6,13,16,14,18,17,7,9,22)$ |
| $C_{6} \cup C_{8} \cup C_{9}$ | $(1,19,16,3,23,11,2,20)(4,14,15,12,21,7,8,17,13)(5,18,6,10,22,9)$ |
| $2 C_{3} \cup C_{8} \cup C_{9}$ | $(1,20,2,11,23,3,13,19)(4,15,10,7,8,21,12,16,14)(5,22,9)(6,18,17)$ |
| $2 C_{7} \cup C_{9}$ | $(1,19,13,15,3,23,11,2,20)(4,10,6,18,17,12,21)(5,22,9,8,7,16,14)$ |
| $C_{3} \cup C_{4} \cup C_{7} \cup C_{9}$ | $(1,19,4,15,3,23,11,2,20)(5,12,21,7,9,22,10)(6,18,17,8)(13,16,14)$ |
| $C_{3} \cup C_{5} \cup C_{6} \cup C_{9}$ | $(1,20,2,11,23,3,15,13,19)(4,12,21,8,7,10)(5,22,9,16,14)(6,18,17)$ |
| $2 C_{4} \cup C_{6} \cup C_{9}$ | $(1,19,4,15,3,23,11,2,20)(5,12,21,10)(6,22,7,9,16,8)(13,14,18,17)$ |

Table 13: Strong VMTLs of the remaining 38 2-regular graphs of order 23

| Graph | Edge Labels |
| :---: | :---: |
| $\mathrm{C}_{4} \cup 2 \mathrm{C}_{5} \cup C_{9}$ | $(1,19,16,13,3,23,11,2,20)(4,14,18,10,15)(5,22,9,21,12)(6,8,7,17)$ |
| $3 C_{3} \cup C_{5} \cup C_{9}$ | $(1,19,16,15,3,23,11,2,20)(4,12,21)(5,9,8,7,22)(6,17,13)(10,14,18)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{9}$ | $(1,19,16,14,3,23,11,2,20)(4,12,21)(5,13,18)(6,8,7,22)(9,15,17,10)$ |
|  | $(1,14,7,21,8,18,16,15)(2,20,3,11,9,4,13,17)(5,19,6,12,23,10,22)$ |
| $C_{3} \cup C_{4} \cup 2 C_{8}$ | (1, 14, 7, 17, 2, 20, 3, 15) (4, 16, 12, 23, 10, 22, 5, 9) (6, 11, 19) (8, 18, 13, 21) |
| $C_{3} \cup C_{5} \cup C_{7} \cup C_{8}$ | (1,15, 3, 20, 2, 17, 14)(4,9, 16)(5, 12, 23, 10, 22)(6, 8, 13, 21, 7, 19, 11, 18) |
| $2 C_{4} \cup C_{7} \cup C_{8}$ | $(1,14,16,18,6,7,11,15)(2,20,3,17)(4,13,8,21)(5,9,19,12,23,10,22)$ |
| $C_{3} \cup 2 C_{6} \cup C_{8}$ | $(1,14,12,23,10,22,5,15)(2,20,3,21,4,17)(6,7,11,19,9,8)(13,18,16)$ |
| $C_{4} \cup C_{5} \cup C_{6} \cup C_{8}$ | $(1,15,16,9,4,14)(2,20,3,17)(5,12,23,10,22)(6,8,13,21,7,19,11,18)$ |
| $3 C_{3} \cup C_{6} \cup C_{8}$ | (1,14, 20, 2, 17, 13, 5, 15) (3, 11, |
| ${ }^{C_{5}} \cup \mathrm{C}_{8}$ | $(1,14,11,3,15)(2,20,6,7,17)(4,19,9,21,8,13,18,16)(5,12,23,10,22)$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{8}$ | $(1,15,16,14)(2,17,3,11,7,18,6,20)(4,19,9)(5,12,23,10,22)(8,13,21)$ |
| $C_{3} \cup 3 C_{4} \cup C_{8}$ | $(1,15,16,14)(2,18,6,17)(3,19,7,11)(4,13,21)(5,8,20,9,12,23,10,22)$ |
| $5 C_{3} \cup C_{8}$ | $(1,15,14)(2,21,4,16,18,3,11,17)(5,13,19)(6,7,20)(8,22,9)(10,12,23)$ |
| $\cup C_{6} \cup 2 C_{7}$ | (1,23, $, 15,17,2,14)(3,19,16,4,10,18)(5,21,9,20,13,12,22)(6,11,7)$ |
| $C_{4} \cup C_{5} \cup 2 C_{7}$ | (1,23, $, 18,17,2,14)(3,19,4,10,15)(5,16,12,22)(6,11,21,9,20,13,7)$ |
|  | $(1,23,8,18,17,2,14)(3,19,15)(4,9,21,6,22,11,10)(5,12,20)(7,16,13)$ |
| $\cup 2 C_{6} \cup C_{7}$ | $(1,23,8,17,2,14)(3,18,15,19)(4,10,13,22,5,12,16)(6,7,11,21,9,20)$ |
| $2 C_{5} \cup C_{6} \cup C_{7}$ | $(1,23,8,17,2,14)(3,18,16,13,19)(4,10,7,21,9)(5,22,11,12,6,20,15)$ |
| $2 C_{3} \cup C_{4} \cup C_{6} \cup C_{7}$ | $(1,23,8,17,2,14)(3,19,15,13,22,5,18)(4,10,16)(6,11,7)(9,20,12,21)$ |
| $2 C_{3} \cup 2 C_{5} \cup C_{7}$ | $(1,23,8,6,17,2,14)(3,19,10)(4,22,12,5,16)(7,11,21,9,18)(13,20,15)$ |
| $C_{3} \cup 2 C_{4} \cup C_{5} \cup C_{7}$ | $(1,23,8,6,17,2,14)(3,19,10)(4,13,5,16)(7,18,9,21)(11,22,12,20,15)$ |
| $4 C_{4} \cup C_{7}$ | $(1,23,8,5,17,2,14)(3,15,11,18)(4,16,7,10)(6,21,9,19)(12,20,13,22)$ |
| $4 C_{3} \cup C_{4} \cup C_{7}$ | $(1,23,8,10,17,2,14)(3,18,15,11)(4,19,16)(5,12,20)(6,22,7)(9,13,21)$ |
| $C_{5} \cup 3 C_{6}$ | $(1,18,12,23,4,22)(2,16,6,8,13,11)(3,14,19,15,17)(5,10,21,7,9,20)$ |
| $2 C_{3} \cup C_{5} \cup 2 C_{6}$ | $(1,18,12,23,4,22)(2,11,3,13,8,16)(5,15,17)(6,9,20,14,19)(7,10,21)$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{6}$ | $(1,18,12,23,4,22)(2,16,13,20,5,11)(3,19,15,17)(6,8,9)(7,21,10,14)$ |
| $C_{3} \cup C_{4} \cup 2 C_{5} \cup C_{6}$ | $(1,18,12,23,4,22)(2,19,6,11)(3,17,15)(5,9,7,21,10)(8,16,13,20,14)$ |
| $3 C_{4} \cup C_{5} \cup C_{6}$ | $(1,18,12,23,4,22)(2,15,3,11)(5,17,16,8,20)(6,14,7,9)(10,19,13,21)$ |
| $4 C_{3} \cup C_{5} \cup C_{6}$ | $(1,22,6,16,18)(2,13,11)(3,17,15)(4,12,23)(5,9,8,21,10,20)(7,14,19)$ |
| $3 C_{3} \cup 2 C_{4} \cup C_{6}$ | $(1,22,7,18)(2,19,3,11)(4,12,23)(5,13,21)(6,14,10,20,8,9)(15,17,16)$ |
| $C_{3} \cup 4 C_{5}$ | $(1,12,4,13,22)(2,17,14,6,16)(3,11,21,8,18)(5,10,23,7,20)(9,15,19)$ |
| $2 C_{4} \cup 3 C_{5}$ | $(1,12,4,13,22)(2,17,5,16)(3,11,20,8,21)(6,14,18,9)(7,23,10,15,19)$ |
| $C_{3} \cup C_{4} \cup 2 C_{5}$ | $(1,12,13,22)(2,17,14)(3,11,21,6,15)(4,18,16)(5,10,23,7,19)(8,9,20)$ |
| $2 C_{3} \cup 3 C_{4} \cup C_{5}$ | $(1,12,19,13,22)(2,15,14)(3,16,18)(4,20,6,21)(5,9,11,17)(7,23,10,8)$ |
| $6 C_{3} \cup C_{5}$ | $(1,22,12)(2,17,14)(3,11,21,4,18)(5,15,13)(6,9,20)(7,23,10)(8,19,16)$ |
| $C_{3} \cup 5 C_{4}$ | $(1,22,6,12)(2,19,13,14)(3,23,8,11)(4,21,9,20)(5,10,7,15)(16,18,17)$ |
| $5 C_{3} \cup 2 C_{4}$ |  |

Table 14: Strong VMTLs of the first 65 2-regular graphs of order 25

| Graph | Edge Labels |
| :---: | :---: |
| $\mathrm{C}_{25}$ | ( $1,24,3,17,18,8,14,2,16,21,12,22,7,10,4,19,13,6,9,15,23,5,25,11,20)$ |
| $C_{3} \cup C_{22}$ | $(1,21,10,25,2,16,22,11,18,3,20,6,8,17,13,4,24,12,7,9,23,14)(5,15,19)$ |
| $C_{4} \cup C_{21}$ | (1,22, 15, 6, 10, 7, 11, 25, 2, 23, 3, 12, 19, 5, 14, 24, 8, 20, 9, 21, 13) (4, 18, 17, 16) |
| $C_{5} \cup C_{20}$ | $(1,23,2,19,15)(3,20,18,8,6,11,25,5,10,22,9,13,7,12,17,16,21,14,4,24)$ |
| $C_{6} \cup C_{19}$ | $(1,19,9,10,22,2,25,13,4,14,7,23,8,21,5,11,3,12,24)(6,16,18,15,20,17)$ |
| $2 C_{3} \cup C_{19}$ | $(1,24,12,3,11,7,23,8,14,20,9,10,6,22,2,25,13,4,19)(5,21,16)(15,18,17)$ |
| $C_{7} \cup C_{18}$ | $(1,15,2,22,3,17,18,16,6,13,23,5,9,21,10,19,7,20)(4,14,24,8,25,12,11)$ |
| $\cup C_{4} \cup C_{18}$ | $(1,15,2,22,3,17,18,4,14,24,8,25,12,7,19,11,16,20)(5,9,6,23)(10,21,13)$ |
| $C_{8} \cup C_{17}$ | $(1,13,24,6,23,8,12,4,18,3,14,20,15,21,17,2,25)(5,19,9,16,7,11,22,10)$ |
|  | $(1,25,2,17,19,15,9,11,21,14,3,18,4,12,6,24,13)(5,20,8,23,10)(7,16,22)$ |
| $2 C_{4} \cup C_{17}$ | $(1,25,2,17,19,4,12,23,8,10,5,15,9,16,6,24,13)(3,14,20,18)(7,22,11,21)$ |
| $C_{9} \cup C_{16}$ | $(1,20,14,12,8,23,13,2,16,22,15,17,6,21,4,18)(3,11,24,9,7,10,19,5,25)$ |
| $C_{3} \cup C_{6} \cup C_{16}$ | $(1,20,7,19,4,18)(2,13,23,8,21,17,15,22,12,5,25,3,11,24,9,16)(6,14,10)$ |
| $C_{4} \cup C_{5} \cup C_{16}$ | $(1,18,4,21,17,10,6,14,9,24,11,3,25,5,12,20)(2,16,8,23,13)(7,22,15,19)$ |
| $3 C_{3} \cup C_{16}$ | $(1,20,18)(2,16,6,19,4,12,8,23,9,24,11,3,25,5,21,13)(7,17,10)(14,22,15)$ |
|  | $(1,20,16,2,21,13,9,17,11,19)(3,14,5,10,22,15,23,12,4,25,8,6,18,7,24)$ |
| $C_{3} \cup C_{7} \cup C_{15}$ | $(1,20,14,3,24,7,18,6,22,10,5,9,13,17,19)(2,21,16)(4,25,8,11,15,23,12)$ |
| $C_{4} \cup C_{6} \cup C_{15}$ | $(1,20,17,19)(2,16,22,8,25,4,12,23,9,5,10,18,6,13,21)(3,24,7,15,11,14)$ |
| $2 C_{5} \cup C_{15}$ | $(1,20,18,8,25,4,12,23,7,24,3,14,11,17,19)(2,16,6,13,21)(5,9,15,22,10)$ |
| ${ }_{3} \cup C_{4} \cup C_{15}$ | $(1,20,6,19)(2,16,22,15,17,18,12,4,25,8,14,3,24,7,21)(5,10,9)(11,13,23)$ |
|  | $(1,22,7,23,3,12,25,13,5,11,9,24,4,18)(2,19,6,8,16,20,14,21,10,17,15)$ |
| $C_{3} \cup C_{8} \cup C_{14}$ | $(1,22,10,17,5,11,7,18)(2,19,15)(3,12,25,13,16,20,4,24,9,21,14,6,8,23)$ |
| $C_{4} \cup C_{7} \cup C_{14}$ | $(1,18,4,24,9,16,15,2,19,17,7,20,10,22)(3,12,25,13,5,11,23)(6,14,21,8)$ |
| $C_{5} \cup C_{6} \cup C_{14}$ | $(1,22,5,11,7,17,15,2,19,16,9,24,4,18)(3,12,25,13,23)(6,14,20,10,21,8)$ |
| $2 C_{3} \cup C_{5} \cup C_{14}$ | $(1,18,7,11,5,17,15,2,19,16,4,24,9,22)(3,12,25,13,23)(6,21,8)(10,20,14)$ |
| $C_{3} \cup 2 C_{4} \cup C_{14}$ | $(1,22,10,20,16,9,24,4,23,3,12,25,13,18)(2,19,15)(5,17,7,11)(6,14,21,8)$ |
|  | $(1,17,15,21,5,16,7,23,6,10,12,2,18)(3,22,9,19,8,25,13,24,11,4,20,14)$ |
| $C_{3} \cup C_{9} \cup C_{13}$ | $(1,17,5,16,10,19,12,2,18)(3,22,6,21,15,8,25,13,24,11,4,20,14)(7,23,9)$ |
| $C_{4} \cup C_{8} \cup C_{13}$ | $(1,17,6,10,16,5,23,14,3,22,12,2,18)(4,20,9,21,15,7,24,11)(8,25,13,19)$ |
| $C_{5} \cup C_{7} \cup C_{13}$ | $(1,17,5,16,7,23,9,22,3,14,12,2,18)(4,20,8,25,13,24,11)(6,10,19,15,21)$ |
| $2 C_{6} \cup C_{13}$ | $(1,17,6,10,21,9,23,14,3,22,12,2,18)(4,20,16,5,24,11)(7,19,8,25,13,15)$ |
| $2 C_{3} \cup C_{6} \cup C_{13}$ | $(1,17,10,6,23,7,24,11,4,20,12,2,18)(3,14,22)(5,16,21)(8,25,13,9,19,15)$ |
| $3 \cup C_{4} \cup C_{5} \cup C_{13}$ | $(1,18,2,12,17)(3,22,10,6,24,11,4,20,7,15,19,9,14)(5,21,16)(8,25,13,23)$ |
| $3 C_{4} \cup C_{13}$ | $(1,17,14,3,22,8,25,13,24,11,12,2,18)(4,23,9,20)(5,10,6,16)(7,19,15,21)$ |
| $4 C_{3} \cup C_{13}$ | $(1,17,6,9,23,4,20,10,21,16,12,2,18)(3,14,22)(5,24,11)(7,15,19)(8,25,13)$ |
| $C_{3} \cup C_{10} \cup C_{12}$ | $(1,17,11,25,9,18,4,12,2,19)(3,22,15,23,8,24,5,10,7,16,14,21)(6,13,20)$ |
| $C_{4} \cup C_{9} \cup$ | $(1,17,13,20,7,16,10,18,4,12,2,19)(3,22,15,23,8,24,5,14,21)(6,11,25,9)$ |
| $C_{5} \cup C_{8} \cup C_{12}$ | $(1,19,2,12,16,14,9,25,11,4,18,17)(3,22,15,23,8,24,5,21)(6,13,20,7,10)$ |
| $C_{6} \cup C_{7} \cup C_{12}$ | $(1,19,2,12,16,11,25,9,22,3,21,17)(4,13,10,6,20,15,18)(5,24,8,7,23,14)$ |
| $2 C_{3} \cup C_{7} \cup C_{12}$ | $(1,17,6,21,9,25,11,15,16,12,2,19)(3,13,22)(4,20,18)(5,14,23,10,7,8,24)$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup C_{12}$ | $(1,17,14,12,2,19)(3,21,16,22)(4,18,5,24,8,11,25,9,6,10,7,23)(13,20,15)$ |
| $C_{3} \cup 2 C_{5} \cup C_{12}$ | $(1,17,10,7,9,25,11,22,3,12,2,19)(4,18,13,6,20)(5,24,8,15,23)(14,16,21)$ |
| $2 C_{4} \cup C_{5} \cup C_{12}$ | $(1,19,2,12,4,18,15,13,22,3,21,17)(5,24,8,23,14)(6,11,25,9)(7,16,10,20)$ |
| $\begin{gathered} 3 C_{3} \cup C_{4} \cup C_{12} \\ C_{3} \cup 2 C_{12} \end{gathered}$ | $(1,17,5,24,8,7,9,25,11,12,2,19)(3,16,14)(4,23,10,21)(6,20,18)(13,22,15)$ <br> $(1,13,24)(2,22,6,12,3,16,5,25,4,18,14)(7,20,15,8,9,11,23,10,21,17,19)$ |
| $C_{4} \cup C_{10}$ | $(1,24,10,21,6,15,23,14,2,22,13)(3,12,20,8,9,11,7,19,17,16)(4,18,5,25)$ |
| $C_{5} \cup C_{9} \cup$ | $(1,24,10,21,7,11,9,8,13)(2,22,5,25,4,19,17,18,15,23,14)(3,12,20,6,16)$ |
| $C_{6} \cup C_{8} \cup C_{11}$ | $(1,24,10,11,21,7,20,13)(2,22,9,8,18,4,25,5,15,23,14)(3,12,6,17,19,16)$ |
| $2 C_{3} \cup C_{8}$ | $(1,13,24)(2,22,14)(3,12,6,17,15,20,18,4,25,5,16)(7,21,10,23,11,9,8,19)$ |
| $2 C_{7} \cup C_{11}$ | $(1,24,10,21,11,7,13)(2,22,5,25,4,19,17,18,15,23,14)(3,16,6,20,8,9,12)$ |
| $C_{3} \cup C_{4} \cup C_{7} \cup C_{11}$ | $(1,13,24)(2,22,6,14)(3,16,5,25,4,18,9,8,15,20,12)(7,11,23,10,21,17,19)$ |
| $C_{3} \cup C_{5} \cup C_{6} \cup C_{11}$ | $(1,13,24)(2,22,11,20,14)(3,16,7,10,8,12)(4,25,5,21,6,15,23,9,19,17,18)$ |
| $2 C_{4} \cup C_{6} \cup C_{11}$ | $(1,24,9,11,21,13)(2,14,23,8,20,15,6,12,3,16,22)(4,18,5,25)(7,19,17,10)$ |
| $C_{4} \cup 2 C_{5} \cup C_{11}$ | $(1,13,21,7,24)(2,22,5,25,4,19,17,18,15,23,14)(3,12,20,6,16)(8,9,11,10)$ |
| $3 C_{3} \cup C_{5} \cup C_{11}$ | $(1,13,24)(2,22,14)(3,16,5,25,4,18,8,20,15,19,12)(6,21,17)(7,10,23,9,11)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{11}$ | $(1,13,24)(2,22,6,15,20,16,3,12,21,17,14)(4,18,5,25)(7,10,8,19)(9,11,23)$ |
| $C_{5} \cup 2 C_{10}$ | $(1,16,15,7,12,6,8,19,2,14)(3,22,4,20,17)(5,18,11,21,9,24,10,25,13,23)$ |
| $C_{6} \cup C_{9} \cup C_{10}$ | $(1,14,2,19,17,3,22,15,16)(4,18,12,11,7,20)(5,23,9,24,10,25,13,6,8,21)$ |
| $2 C_{3} \cup C_{9} \cup C_{10}$ | $(1,14,2,19,11,17,6,18,4,16)(3,22,15)(5,9,24,10,25,13,23,8,21)(7,12,20)$ |
| $C_{7} \cup C_{8} \cup C_{10}$ | $(1,14,2,19,18,5,23,4,20,16)(3,22,9,24,10,25,13,17)(6,12,7,15,11,21,8)$ |
| $C_{3} \cup C_{4} \cup C_{8} \cup C_{10}$ | $(1,16,6,8,18,19,2,14)(3,17,7,22)(4,15,21,9,24,10,25,13,5,23)(11,12,20)$ |
| $C_{3} \cup C_{5} \cup C_{7} \cup C_{10}$ | $(1,14,2,19,9,24,10,25,13,16)(3,22,5,18,4,20,17)(6,12,7,23,8)(11,21,15)$ |
| $2 C_{4} \cup C_{7} \cup C_{10}$ | $(1,14,2,19,4,18,12,15,21,16)(3,22,7,17)(5,23,9,24,10,25,13)(6,8,11,20)$ |
| $C_{3} \cup 2 C_{6} \cup C_{10}$ | $(1,14,2,19,8,18,4,15,21,16)(3,22,6,23,7,17)(5,9,24,10,25,13)(11,12,20)$ |
| $C_{4} \cup C_{5} \cup C_{6} \cup C_{10}$ | $(1,16,21,7,11,18,12,19,2,14)(3,22,5,17)(4,20,6,8,15)(9,24,10,25,13,23)$ |

Table 15: Strong VMTLs of the remaining 65 2-regular graphs of order 25.

| Graph | Edge Labels |
| :---: | :---: |
| $3 C_{3} \cup C_{6} \cup C_{10}$ | (1, 14, 2, 19, 4, 23, 9, 11, 20, 16)(3, 22, 15)(5, 24, 10, 25, 13, 17)(6, 8, 18)(7, 21, 12) |
| $3 C_{5} \cup C_{10}$ | $(1,16,21,15,4,18,12,19,2,14)(3,23,9,11,22)(5,24,10,25,13)(6,17,7,20,8)$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{10}$ | $(1,14,2,19,17,20,4,22,3,16)(5,18,12,15)(6,23,8)(7,21,11)(9,24,10,25,13)$ |
| $C_{3} \cup 3 C_{4} \cup C_{10}$ | $(1,14,2,19,12,6,21,8,20,16)(3,11,22)(4,18,5,15)(7,23,9,17)(10,25,13,24$ |
| $5 C_{3} \cup C_{10}$ | $(1,16,14)(2,24,10,25,13,3,18,6,17,20)(4,15,21)(5,23,9)(7,22,11)(8,19,12)$ |
| $C_{7} \cup 2 C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,22,9,23,14,19,7,20)(6,10,13,8,21,17,18)$ |
| $C_{3} \cup C_{4} \cup 2 C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,22,9,23,6,10,13,20)(7,14,21,17)(8,19,18)$ |
| $2 C_{8} \cup C_{9}$ | $(1,16,3,15,5,25,4,24)(2,20,7,8,18,6,10,11,12)(9,22,13,21,17,19,14,23)$ |
| $C_{3} \cup C_{5} \cup C_{8} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,22,7,19,8,13,20)(6,10,21,17,18)(9,14,23)$ |
| $2^{2} C_{4} \cup C_{8} \cup C_{9}$ | $(1,16,3,15,5,25,4,24)(2,20,18,19,8,13,22,14,12)(6,9,7,17)(10,21,11,23)$ |
| $C_{3} \cup C_{6} \cup C_{7} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,20)(6,18,19,8,23,10)(7,22,13,21,17,9,14)$ |
| $C_{4} \cup C_{5} \cup C_{7} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,22,7,20)(6,10,23,8,13,19,18)(9,14,21,17)$ |
| $3 C_{3} \cup C_{7} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,20)(6,23,10)(7,14,9,22,13,21,17)(8,18,19)$ |
| $C_{4} \cup 2 C_{6} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,20,7,9,22,12)(6,23,10,14,21,17)(8,18,19,13)$ |
| $2 C_{5} \cup C_{6} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,22,10,13,20)(6,18,19,8,23)(7,14,21,17,9)$ |
| $2 C_{3} \cup C_{4} \cup C_{6} \cup C_{9}$ | $(1,16,3,15,5,25,11,4,24)(2,12,23,10,14,20)(6,21,17)(7,9,22)(8,13,19,18)$ |
| $2 C_{3} \cup 2 C_{5} \cup C_{9}$ | $(1,16,3,15,23,9,22,4,24)(2,12,21,14,20)(5,25,11)(6,17,10,19,18)(7,13,8)$ |
| ${ }_{3} \cup 2 C_{4} \cup C_{5} \cup C_{9}$ | $(1,16,3,15,12,2,20,4,24)(5,10,23,11,25)(6,14,21,17)(7,22,9)(8,13,19,18)$ |
| $4 C_{4} \cup C_{9}$ | $(1,16,3,15,5,25,13,8,24)(2,20,4,12)(6,9,18,17)(7,21,10,19)(11,22,14,23)$ |
| $4 C_{3} \cup C_{4} \cup C_{9}$ | $(1,16,3,24)(2,12,8,13,10,25,5,11,20)(4,14,22)(6,23,9)(7,21,17)(15,19,18)$ |
| $C_{3} \cup C_{6} \cup 2 C_{8}$ | $(1,17,19,4,12,22,3,16)(2,24,13,9,5,25)(6,14,10,11,21,7,8,23)(15,20,18)$ |
| $C_{4} \cup C_{5} \cup 2 C_{8}$ | $(1,16,3,22,14,20,18,17)(2,24,13,7,9,19,5,25)(4,11,21,12,10)(6,15,8,23)$ |
| $3 C_{3} \cup 2 C_{8}$ | $(1,16,17)(2,24,13,22,3,19,5,25)(4,10,6,23,8,7,21,15)(9,11,12)(14,20,18)$ |
| $C_{3} \cup 2 C_{7} \cup C_{8}$ | $(1,16,3,22,14,7,17)(2,25,5,11,21,13,24)(4,19,9,6,23,8,12,10)(15,20,18)$ |
| $\cup C_{6} \cup C_{7} \cup C_{8}$ | $(1,16,3,22,12,20,18,17)(2,24,13,9,11,5,25)(4,19,14,10)(6,15,21,7,8,23)$ |
| $2 C_{5} \cup C_{7} \cup C_{8}$ | $(1,16,3,19,17)(2,24,13,7,9,5,25)(4,11,10,14,21)(6,22,12,20,18,15,8,23)$ |
| $2 C_{3} \cup C_{4} \cup C_{7} \cup C_{8}$ | $(1,17,3,16)(2,24,13,19,15,10,5,25)(4,20,18)(6,8,23)(7,21,14,22,11,12,9)$ |
| $C_{5} \cup 2 C_{6} \cup C_{8}$ | $(1,16,3,22,10,4,19,17)(2,24,13,11,5,25)(6,23,8,12,9)(7,21,14,20,18,15)$ |
| $2 C_{3} \cup C_{5} \cup C_{6} \cup C_{8}$ | $(1,16,20,14,21,17)(2,24,13,7,9,19,5,25)(3,12,11,10,22)(4,15,18)(6,23,8)$ |
| $C_{3} \cup 2 C_{4} \cup C_{6} \cup C_{8}$ | $(1,17,3,16)(2,24,13,11,22,10,5,25)(4,12,9,19)(6,23,8)(7,15,21,14,20,18)$ |
| ${ }_{3} \cup C_{4} \cup 2 C_{5} \cup C_{8}$ | $(1,17,3,16)(2,24,13,9,14,19,5,25)(4,12,22,10,11)(6,8,23)(7,21,15,20,18)$ |
| $3 C_{4} \cup C_{5} \cup C_{8}$ | $(1,17,19,16)(2,24,13,21,7,9,5,25)(3,12,11,10,22)(4,20,18,15)(6,14,8,23)$ |
| $4 C_{3} \cup C_{5} \cup C_{8}$ | $(1,16,17)(2,24,13,7,9,19,5,25)(3,20,18)(4,15,21,14,11)(6,8,23)(10,12,22)$ |
| $3 C_{3} \cup 2 C_{4} \cup C_{8}$ | $(1,16,17)(2,24,13,23,8,14,5,25)(3,12,22)(4,10,19)(6,15,20,18)(7,9,11,21)$ |
| $C_{4} \cup 3 C_{7}$ | $(1,17,18,6,16,4,24)(2,19,14,13)(3,11,25,7,9,10,20)(5,12,22,15,23,8,21)$ |
| $C_{5} \cup C_{6} \cup 2 C_{7}$ | $(1,17,18,16,4,24)(2,19,14,12,5,9,13)(3,21,6,10,20)(7,22,15,23,8,11,25)$ |
| $2 C_{3} \cup C_{5} \cup 2 C_{7}$ | $(1,17,16,4,24)(2,13,22,15,23,8,19)(3,14,20)(5,9,21)(6,18,11,25,7,12,10)$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{7}$ | $(1,17,10,6,18,4,24)(2,19,16,13)(3,14,20)(5,9,11,25,7,12,21)(8,22,15,23)$ |
| $3 C_{6} \cup C_{7}$ | $(1,17,3,20,4,24)(2,19,14,21,6,16,13)(5,11,25,7,10,9)(8,18,12,22,15,23)$ |
| $2 C_{3} \cup 2 C_{6} \cup C_{7}$ | $(1,24,4,12,10,17)(2,19,11,25,7,22,13)(3,21,16)(5,18,15,23,8,9)(6,14,20)$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{6} \cup C_{7}$ | $(1,17,3,20,4,24)(2,13,21,14,19)(5,9,18,12)(6,10,16)(7,22,15,23,8,11,25)$ |
| $3 C_{4} \cup C_{6} \cup C_{7}$ | $(1,17,12,10,4,24)(2,19,16,3,14,20,13)(5,21,6,18)(7,9,11,25)(8,22,15,23)$ |
| $4 C_{3} \cup C_{6} \cup C_{7}$ | $(1,24,4,11,25,7,17)(2,19,16,6,8,18)(3,13,20)(5,12,22)(9,10,21)(14,15,23)$ |
| $C_{3} \cup 3 C_{5} \cup C_{7}$ | $(1,17,10,4,24)(2,19,18,5,21,3,13)(6,16,14)(7,12,22,11,25)(8,9,20,15,23)$ |
| $2 C_{4} \cup 2 C_{5} \cup C_{7}$ | $(1,17,10,4,24)(2,19,3,16,13)(5,12,21,14,20,6,18)(7,9,11,25)(8,22,15,23)$ |
| ${ }_{3} \cup C_{4} \cup C_{5} \cup C_{7}$ | $(1,24,4,18,17)(2,19,14,13)(3,21,16)(5,12,22,7,25,11,9)(6,10,20)(8,15,23)$ |
| $2 C_{3} \cup 3 C_{4} \cup C_{7}$ | $(1,17,5,14,10,4,24)(2,19,18)(3,12,21,13)(6,20,15,23)(7,16,11,25)(8,22,9)$ |
| $6 C_{3} \cup C_{7}$ | $(1,17,19)(2,13,22)(3,14,20)(4,24,5,25,6,8,18)(7,12,9)(10,15,23)(11,21,16)$ |
| $C_{3} \cup C_{4} \cup 3 C_{6}$ | $(1,16,22)(2,20,4,15,11,23)(3,18,17,10,5,13)(6,8,21,9,19,14)(7,24,12,25)$ |
| $C_{3} \cup 2 C_{5} \cup 2 C_{6}$ | $(1,22,14,19,16)(2,23,5,9,20)(3,18,6,21,17,13)(4,15,11)(7,24,10,8,12,25)$ |
| $2 C_{4} \cup C_{5} \cup 2 C_{6}$ | $(1,16,18,3,13,22)(2,20,8,6,23)(4,14,19,11,9,15)(5,21,17,10)(7,24,12,25)$ |
| $3 C_{3} \cup C_{4} \cup 2 C_{6}$ | $(1,16,22)(2,20,6,8,10,23)(3,21,13)(4,11,19,9,18,17)(5,15,14)(7,24,12,25)$ |
| $C_{4} \cup 3 C_{5} \cup C_{6}$ | $(1,22,8,10,16)(2,20,15,4,23)(3,13,21,17,11,18)(5,9,6,14,19)(7,24,12,25)$ |
| $3 C_{3} \cup 2 C_{5} \cup C_{6}$ | $(1,16,22)(2,20,15,5,23)(3,21,13)(4,14,19,17,10)(6,24,7,25,12,9)(8,11,18)$ |
| $C_{3} \cup 2 C_{4} \cup C_{5} \cup C_{6}$ | $(1,16,22)(2,23,5,10,20)(3,18,17,9,11,13)(4,15,19,14)(6,21,8)(7,24,12,25)$ |
| $C_{3} \cup 4 C_{4} \cup C_{6}$ | $(1,22,8,11,18,16)(2,20,15,23)(3,21,5,13)(4,10,17)(6,9,19,14)(7,24,12,25)$ |
| $5 C_{3} \cup C_{4} \cup C_{6}$ | $(1,16,22)(2,23,4,11,18,17)(3,19,15)(5,21,9)(6,14,10)(7,24,12,25)(8,13,20)$ |
|  | $(1,16,6,25,13)(2,22,3,23,14)(4,24,8,10,11)(5,18,17,19,15)(7,12,21,9,20)$ |
| $2 C_{3} \cup C_{4} \cup 3 C_{5}$ | $(1,16,6,25,13)(2,21,12,18,17)(3,22,14,23)(4,24,8,10,11)(5,15,19)(7,9,20)$ |
| $C_{3} \cup 3 C_{4} \cup 2 C_{5}$ | $(1,16,6,25,13)(2,19,17)(3,21,14,23)(4,24,8,22,12)(5,18,15,10)(7,20,9,11)$ |
| $5 C_{3} \cup 2 C_{5}$ | $(1,16,6,25,13)(2,19,14)(3,22,15)(4,23,11)(5,18,17,7,21)(8,12,24)(9,10,20)$ |
| $5 C_{4} \cup C_{5}$ | $(1,16,6,25,13)(2,21,3,18)(4,11,14,12)(5,22,7,23)(8,10,9,24)(15,19,17,20)$ |
| $4 C_{3} \cup 2 C_{4} \cup C_{5}$ | $(1,16,6,25,13)(2,19,17)(3,12,23)(4,21,8,24)(5,22,15,18)(7,9,11)(10,20,14)$ |
| $3 C_{3} \cup 4 C_{4}$ | $(1,20,17)(2,18,12)(3,16,13,14)(4,21,5,19)(6,22,10)(7,15,23,8)(9,24,11,25)$ |
| $7 C_{3} \cup C_{4}$ | $(1,20,5,17)(2,12,21)(3,14,24)(4,22,15)(6,10,18)(7,23,8)(9,11,25)(13,19,16)$ |

Table 16: Strong VMTLs of the first 70 two-regular graphs of order 27.

|  |  |
| ---: | ---: |
|  | $G r a p h$ |

Table 17: Strong VMTLs of the next 70 two-regular graphs of order 27.

| Graph | Edge Labels |
| :---: | :---: |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{12}$ | $(1,21,3,23,2,13,24,7,9,25,15,26)(4,19,16,17)(5,27,12)(6,14,22)(8,11,18,20,10)$ |
| $C_{3} \cup 3 C_{4} \cup C_{12}$ | $(1,21,3,23,2,13,8,20,17,19,15,26)(4,14,6,25)(5,27,12,18)(7,10,9)(11,22,16,24)$ |
| $5 C_{3} \cup C_{12}$ | $(1,21,3,23,2,13,5,27,12,4,15,26)(6,14,22)(7,16,24)(8,9,25)(10,11,19)(17,20,18)$ |
| $C_{5} \cup 2 C_{11}$ | $(1,25,3,21,20)(2,15,10,12,7,23,4,19,17,22,16)(5,26,8,27,13,24,9,6,14,18,11)$ |
| $C_{6} \cup C_{10} \cup C_{11}$ | $(1,25,3,17,22,16,2,15,21,20)(4,23,7,12,10,19)(5,26,8,27,13,24,9,6,18,14,11)$ |
| $2 C_{3} \cup C_{10} \cup C_{11}$ | $(1,20,16,2,15,12,7,22,3,25)(4,11,19)(5,17,21,18,23,9,24,13,27,8,26)(6,10,14)$ |
| $C_{7} \cup C_{9} \cup C_{11}$ | $(1,25,3,12,18,21,20)(2,15,7,17,19,4,23,6,10,22,16)(5,26,8,27,13,24,9,11,14)$ |
| $C_{3} \cup C_{4} \cup C_{9} \cup C_{11}$ | $(1,20,3,25)(2,15,12,4,11,21,17,22,14,6,16)(5,26,8,27,13,24,9,10,19)(7,18,23)$ |
| $2 C_{8} \cup C_{11}$ | $(1,20,19,4,12,17,3,25)(2,16,22,14,11,21,6,18,23,7,15)(5,26,8,27,13,24,9,10)$ |
| $C_{3} \cup C_{5} \cup C_{8} \cup C_{11}$ | $(1,20,12,3,25)(2,16,6,21,17,22,14,10,19,4,15)(5,26,8,27,13,24,9,11)(7,18,23)$ |
| $2 C_{4} \cup C_{8} \cup C_{11}$ | $(1,20,3,25)(2,15,23,16)(4,21,11,19,22,14,6,18)(5,26,8,27,13,24,9,7,12,17,10)$ |
| $C_{3} \cup C_{6} \cup C_{7} \cup C_{11}$ | $(1,20,12,17,21,3,25)(2,15,7,18,23,16)(4,11,19)(5,26,8,27,13,24,9,10,6,14,22)$ |
| $C_{4} \cup C_{5} \cup C_{7} \cup C_{11}$ | $(1,20,3,25)(2,15,14,6,18,23,16)(4,11,19,17,21)(5,26,8,27,13,24,9,7,12,10,22)$ |
| $3 C_{3} \cup C_{7} \cup C_{11}$ | $(1,20,19,4,12,3,25)(2,15,21,17,24,13,27,8,26,5,22)(6,16,14)(7,11,18)(9,10,23)$ |
| $C_{4} \cup 2 C_{6} \cup C_{11}$ | $(1,20,3,25)(2,15,23,6,14,16)(4,11,5,26,8,27,13,24,9,18,21)(7,17,19,22,10,12)$ |
| $2 C_{5} \cup C_{6} \cup C_{11}$ | $(1,20,12,3,25)(2,15,4,23,7,16)(5,26,8,27,13,24,9,11,18,21,17)(6,19,22,14,10)$ |
| ${ }_{3} \cup C_{4} \cup C_{6} \cup C_{11}$ | $(1,20,2,15,3,25)(4,11,5,26,8,27,13,24,9,23,16)(6,17,21)(7,18,12)(10,19,22,14)$ |
| $2 C_{3} \cup 2 C_{5} \cup C_{11}$ | $(1,20,12,3,25)(2,17,5,26,8,27,13,24,9,21,15)(4,19,22,16,23)(6,10,14)(7,11,18)$ |
| $\cup 2 C_{4} \cup C_{5} \cup C_{11}$ | $(1,20,3,25)(2,15,23,16)(4,11,21)(5,26,8,27,13,24,9,18,12,7,17)(6,10,19,22,14)$ |
| $4 C_{4} \cup C_{11}$ | $(1,20,3,25)(2,14,10,15)(4,18,23,16)(5,26,8,27,13,24,9,6,12,7,22)(11,19,17,21)$ |
| $4 C_{3} \cup C_{4} \cup C_{11}$ | $(1,20,7,25)(2,23,15)(3,16,21)(4,14,6,10,5,26,8,27,13,9,24)(11,18,12)(17,19,22)$ |
| $C_{7} \cup 2 C_{10}$ | $(1,23,2,24,15,26,14,22,16,19)(3,20,11,17,10,8,21,13,4,18)(5,25,12,7,9,6,27)$ |
| $C_{3} \cup C_{4} \cup 2 C_{10}$ | $(1,23,2,24,13,4,18,3,20,19)(5,27,6,10,17,12,16,22,9,25)(7,11,8)(14,21,15,26)$ |
| $C_{8} \cup C_{9} \cup C_{10}$ | $(1,23,2,24,15,26,14,21,17,19)(3,20,8,7,11,16,13,4,18)(5,25,12,22,9,10,6,27)$ |
| $C_{3} \cup C_{5} \cup C_{9} \cup C_{10}$ | $(1,19,17,22,16,11,24,2,23)(3,20,8,7,9,10,21,13,4,18)(5,27,6,12,25)(14,15,26)$ |
| $2 C_{4} \cup C_{9} \cup C_{10}$ | $(1,23,2,24,11,4,18,3,20,19)(5,25,6,27)(7,12,16,13,21,17,10,8,9)(14,22,15,26)$ |
| $C_{3} \cup C_{6} \cup C_{8} \cup C_{10}$ | $(1,23,2,24,12,19)(3,20,8,9,7,11,4,18)(5,25,10,17,22,16,21,13,6,27)(14,15,26)$ |
| $C_{4} \cup C_{5} \cup C_{8} \cup C_{10}$ | $(1,19,10,8,9,7,12,24,2,23)(3,20,17,22,16,11,4,18)(5,25,6,27)(13,15,26,14,21)$ |
| $3 C_{3} \cup C_{8} \cup C_{10}$ | $(1,19,17,22,13,21,7,24,2,23)(3,18,20)(4,11,16,6,27,5,25,12)(8,9,10)(14,15,26)$ |
| $C_{3} \cup 2 C_{7} \cup C_{10}$ | $(1,19,10,21,17,22,13,24,2,23)(3,18,4,15,26,14,20)(5,25,11,16,12,6,27)(7,9,8)$ |
| $C_{4} \cup C_{6} \cup C_{7} \cup C_{10}$ | $(1,23,2,24,15,26,14,21,17,19)(3,18,4,13,16,11,20)(5,25,12,22,6,27)(7,9,10,8)$ |
| $2 C_{5} \cup C_{7} \cup C_{10}$ | $(1,23,2,24,10,8,9,7,12,19)(3,20,17,21,18)(4,11,16,6,27,5,25)(13,15,26,14,22)$ |
| $2 C_{3} \cup C_{4} \cup C_{7} \cup C_{10}$ | $(1,23,2,24,12,4,18,3,20,19)(5,27,6,13,22,9,25)(7,10,8)(11,16,21,17)(14,15,26)$ |
| $C_{5} \cup 2 C_{6} \cup C_{10}$ | $(1,23,2,24,11,7,8,9,10,19)(3,20,16,12,4,18)(5,25,13,21,6,27)(14,17,22,15,26)$ |
| $2 C_{3} \cup C_{5} \cup C_{6} \cup C_{10}$ | $(1,19,17,10,12,4,13,24,2,23)(3,15,26,14,21,18)(5,27,6,9,25)(7,16,22)(8,20,11)$ |
| $C_{3} \cup 2 C_{4} \cup C_{6} \cup C_{10}$ | $(1,23,2,24,12,4,18,3,20,19)(5,25,6,27)(7,11,17,10,9,8)(13,22,16,21)(14,15,26)$ |
| $C_{3} \cup C_{4} \cup 2 C_{5} \cup C_{10}$ | $(1,19,4,12,15,26,14,24,2,23)(3,18,16)(5,25,11,6,27)(7,21,10,8)(9,20,17,22,13)$ |
| $3 C_{4} \cup C_{5} \cup C_{10}$ | $(1,23,2,24,13,4,18,3,20,19)(5,25,6,27)(7,9,10,8)(11,17,12,22,16)(14,21,15,26)$ |
| $4 C_{3} \cup C_{5} \cup C_{10}$ | $(1,23,2,24,6,27,5,11,20,19)(3,25,12)(4,18,17)(7,16,22)(8,9,10)(13,21,15,26,14)$ |
| $3 C_{3} \cup 2 C_{4} \cup C_{10}$ | $(1,19,9,22,5,27,6,24,2,23)(3,13,16)(4,17,21,18)(7,10,8)(11,12,25)(14,20,15,26)$ |
| $3 C_{9}$ | $(1,18,3,12,21,15,20,4,22)(2,26,5,17,24,16,9,25,14)(6,11,27,10,8,19,13,7,23)$ |
| $C_{3} \cup C_{6} \cup 2 C_{9}$ | $(1,22,11,27,10,8,12,3,18)(2,14,25,9,16,6,24,5,26)(4,13,19,7,17,23)(15,21,20)$ |
| $C_{4} \cup C_{5} \cup 2 C_{9}$ | $(1,22,13,17,23,6,12,3,18)(2,26,5,19,8,24,9,25,14)(4,21,20,16)(7,15,11,27,10)$ |
| $3 C_{3} \cup 2 C_{9}$ | $(1,18,22)(2,26,5,12,3,24,9,25,14)(4,17,15,11,27,10,8,21,20)(6,19,16)(7,13,23)$ |
| $C_{3} \cup C_{7} \cup C_{8} \cup C_{9}$ | $(1,18,3,12,8,24,16,19,22)(2,14,25,9,17,5,26)(4,13,20)(6,23,7,11,27,10,15,21)$ |
| $C_{4} \cup C_{6} \cup C_{8} \cup C_{9}$ | $(1,22,4,20,13,12,3,18)(2,14,25,9,23,7,15,5,26)(6,11,27,10,8,21)(16,19,17,24)$ |
| $2 C_{5} \cup C_{8} \cup C_{9}$ | $(1,18,3,12,8,24,16,19,22)(2,14,25,9,17,13,5,26)(4,21,15,7,20)(6,11,27,10,23)$ |
| $2 C_{3} \cup C_{4} \cup C_{8} \cup C_{9}$ | $(1,18,3,12,8,10,27,11,22)(2,26,5,21,20,9,25,14)(4,13,23)(6,19,16,24)(7,17,15)$ |
| $C_{4} \cup 2 C_{7} \cup C_{9}$ | $(1,18,3,12,23,4,22)(2,14,25,9,15,5,26)(6,16,24,8,21,20,13,17,19)(7,11,27,10)$ |
| $C_{5} \cup C_{6} \cup C_{7} \cup C_{9}$ | $(1,22,8,12,3,18)(2,14,25,9,13,5,26)(4,21,15,17,24,16,19,7,20)(6,11,27,10,23)$ |
| $2 C_{3} \cup C_{5} \cup C_{7} \cup C_{9}$ | $(1,18,22)(2,14,25,9,24,5,26)(3,19,8,16,10,27,11,6,12)(4,21,20,15,17)(7,13,23)$ |
| $C_{3} \cup 2 C_{4} \cup C_{7} \cup C_{9}$ | $(1,22,18)(2,14,25,9,24,5,26)(3,12,10,27,11,6,21,20,15)(4,16,8,17)(7,19,13,23)$ |
| $3 C_{6} \cup C_{9}$ | $(1,22,8,12,3,18)(2,26,5,19,16,24,9,25,14)(4,21,20,6,23,13)(7,11,27,10,17,15)$ |
| $2 C_{3} \cup 2 C_{6} \cup C_{9}$ | $(1,18,22)(2,14,25,9,15,3,12,5,26)(4,21,20,6,16,17)(7,13,23)(8,24,11,27,10,19)$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{6} \cup C_{9}$ | $(1,22,8,12,3,18)(2,14,25,9,23,6,21,5,26)(4,13,20)(7,11,27,10,15)(16,19,17,24)$ |
| $3 C_{4} \cup C_{6}$ | $(1,18,17,23,4,22)(2,14,25,9,24,6,15,5,26)(3,19,13,12)(7,11,27,10)(8,21,20,16)$ |
| $4 C_{3} \cup C_{6} \cup C_{9}$ | $(1,18,22)(2,14,25,10,27,11,16,5,26)(3,17,13,23,6,12)(4,21,20)(7,19,15)(8,9,24)$ |
| $C_{3} \cup 3 C_{5} \cup C_{9}$ | $(1,18,23,13,22)(2,14,25,9,8,21,19,5,26)(3,24,6,20,12)(4,17,16)(7,11,27,10,15)$ |
| $2 C_{4} \cup 2 C_{5} \cup C_{9}$ | $(1,22,7,18)(2,26,5,21,19,8,9,25,14)(3,12,20,15)(4,17,24,6,16)(10,27,11,13,23)$ |
| $3 C_{3} \cup C_{4} \cup C_{5} \cup C_{9}$ | $(1,18,22)(2,14,25,9,12,3,19,5,26)(4,13,16)(6,20,15,21)(7,11,27,10,23)(8,17,24)$ |
| $2 C_{3} \cup 3 C_{4} \cup C_{9}$ | $(1,22,4,18)(2,14,25,9,16,20,15,5,26)(3,21,12)(6,23,17,24)(7,11,27,10)(8,19,13)$ |
| $6 C_{3} \cup C_{9}$ | $(1,18,16)(2,26,5,15,3,24,12,4,21)(6,9,20)(7,17,23)(8,14,25)(10,27,11)(13,19,22)$ |
| $C_{3} \cup 3 C_{8}$ | $(1,19,18,21,17,24,8,27)(2,25,15,9,12,22,3,20)(4,11,5,26,7,10,16,14)(6,13,23)$ |
| $C_{4} \cup C_{7} \cup 2 C_{8}$ | $(1,19,7,26,5,24,8,27)(2,20,3,22,17,21,15,25)(4,13,11,10,9,6,12)(14,16,18,23)$ |
| $C_{5} \cup C_{6} \cup 2 C_{8}$ | $(1,19,13,4,14,16,8,27)(2,20,3,22,15,25)(5,26,7,12,24)(6,9,17,21,18,23,11,10)$ |
| $2 C_{3} \cup C_{5} \cup 2 C_{8}$ | $(1,19,7,26,5,10,8,27)(2,20,3,22,14,24,15,25)(4,12,17)(6,13,11)(9,21,16,18,23)$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{8}$ | $(1,19,7,26,5,24,8,27)(2,20,3,22,17,21,15,25)(4,13,11)(6,10,9,12)(14,16,18,23)$ |
| $C_{5} \cup 2 C_{7} \cup C_{8}$ | $(1,19,13,4,11,8,27)(2,20,3,22,14,23,15,25)(5,26,7,17,24)(6,12,9,21,18,16,10)$ |
| $2 C_{6} \cup C_{7} \cup C_{8}$ | $(1,19,11,13,24,8,27)(2,20,3,22,17,21,15,25)(4,14,7,26,5,12)(6,9,10,16,18,23)$ |
| $2 C_{3} \cup C_{6} \cup C_{7} \cup C_{8}$ | $(1,19,13,4,11,8,27)(2,20,3,22,15,25)(5,26,7,14,10,6,12,24)(9,17,21)(16,18,23)$ |

Table 18: Strong VMTLs of the remaining 51 two-regular graphs of order 27.

| Graph | Edge Labels |
| :---: | :---: |
| $C_{3} \cup C_{4} \cup C_{5} \cup C_{7} \cup C_{8}$ | $(1,19,17,9,21,8,27)(2,20,3,22,10,6,15,25)(4,14,24,13,11)(5,12,7,26)(16,18,23)$ |
| $3 C_{4} \cup C_{7} \cup C_{8}$ | $(1,19,13,6,9,8,27)(2,20,3,22,4,17,12,25)(5,26,7,11)(10,14,16,24)(15,21,18,23)$ |
| $4 C_{3} \cup C_{7} \cup C_{8}$ | $(1,19,11,5,10,8,27)(2,20,3,22,4,13,6,25)(7,26,14)(9,15,23)(12,17,24)(16,21,18)$ |
| $C_{3} \cup C_{4} \cup 2 C_{6} \cup C_{8}$ | $(1,19,11,21,18,16,8,27)(2,20,3,22,15,25)(4,12,9,17,24,14)(5,10,7,26)(6,13,23)$ |
| $C_{3} \cup 2 C_{5} \cup C_{6} \cup C_{8}$ | $(1,19,10,8,27)(2,20,3,22,15,25)(4,13,11,21,17)(5,26,7,9,6,24,12,14)(16,18,23)$ |
| $2 C_{4} \cup C_{5} \cup C_{6} \cup C_{8}$ | $(1,19,11,8,27)(2,20,3,22,17,21,15,25)(4,14,7,26,5,12)(6,18,23,9)(10,16,13,24)$ |
| $3 C_{3} \cup C_{4} \cup C_{6} \cup C_{8}$ | $(1,19,10,6,12,24,8,27)(2,20,3,22,15,25)(4,13,11)(5,26,7,14)(9,17,21)(16,18,23)$ |
| $C_{4} \cup 3 C_{5} \cup C_{8}$ | $(1,19,13,8,27)(2,20,3,22,14,23,15,25)(4,11,6,12)(5,26,7,17,24)(9,21,18,16,10)$ |
| $3 C_{3} \cup 2 C_{5} \cup C_{8}$ | $(1,19,22,8,27)(2,25,13,11,4,14,3,20)(5,24,12,7,26)(6,10,15)(9,17,23)(16,21,18)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{5} \cup C_{8}$ | $(1,19,15,21,4,13,8,27)(2,25,12,3,20)(5,11,7,26)(6,18,23)(9,17,22,10)(14,16,24)$ |
| $C_{3} \cup 4 C_{4} \cup C_{8}$ | $(1,19,4,12,5,21,8,27)(2,20,16,25)(3,18,22)(6,9,10,24)(7,26,13,11)(14,17,15,23)$ |
| $5 C_{3} \cup C_{4} \cup C_{8}$ | $(1,27,8,11,25,2,20,19)(3,18,22)(4,14,12)(5,10,7,26)(6,17,24)(9,15,23)(13,16,21)$ |
| $C_{6} \cup 3 C_{7}$ | $(1,20,3,22,7,27)(2,13,17,15,26,12,24)(4,14,23,10,25,6,16)(5,11,8,9,18,21,19)$ |
| $2 C_{3} \cup 3 C_{7}$ | $(1,27,7,17,22,3,20)(2,13,23,10,9,8,24)(4,14,6,25,5,11,18)(12,15,26)(16,21,19)$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup 2 C_{7}$ | $(1,20,3,22,7,27)(2,13,24)(4,15,26,12,10,8,16)(5,11,6,25)(9,18,21,14,19,17,23)$ |
| $C_{3} \cup 2 C_{5} \cup 2 C_{7}$ | $(1,27,7,11,22,3,20)(2,24,8,9,13)(4,16,23)(5,25,6,10,14)(12,17,18,19,21,15,26)$ |
| $2 C_{4} \cup C_{5} \cup 2 C_{7}$ | $(1,20,3,22,10,7,27)(2,13,9,24)(4,23,14,16)(5,11,8,21,18,17,19)(6,25,15,26,12)$ |
| $3 C_{3} \cup C_{4} \cup 2 C_{7}$ | $(1,20,3,16,23,7,27)(2,13,24)(4,14,18)(5,22,11,9,15,26,12)(6,10,25)(8,21,19,17)$ |
| $C_{3} \cup C_{5} \cup 2 C_{6} \cup C_{7}$ | $(1,20,3,22,7,27)(2,24,8,9,13)(4,14,5,11,19,16)(6,18,21,15,26,12,25)(10,17,23)$ |
| $2 C_{4} \cup 2 C_{6} \cup C_{7}$ | $(1,20,3,22,7,27)(2,13,14,23,9,24)(4,15,26,12,10,8,16)(5,25,6,11)(17,18,21,19)$ |
| $C_{4} \cup 2 C_{5} \cup C_{6} \cup C_{7}$ | $(1,20,3,22,7,27)(2,24,6,25,11,5,13)(4,16,21,18)(8,9,10,14,19)(12,23,17,15,26)$ |
| $3 C_{3} \cup C_{5} \cup C_{6} \cup C_{7}$ | $(1,20,3,22,7,27)(2,13,24)(4,14,16)(5,19,17)(6,25,15,26,12,23,10)(8,11,21,18,9)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{6} \cup C_{7}$ | $(1,20,3,22,7,27)(2,13,24)(4,16,19,11,5,17,23)(6,25,14,18)(8,9,10)(12,21,15,26)$ |
| $4 C_{5} \cup C_{7}$ | $(1,20,3,17,23,7,27)(2,13,22,5,24)(4,14,25,11,21)(6,10,9,8,16)(12,19,18,15,26)$ |
| $2 C_{3} \cup C_{4} \cup 2 C_{5} \cup C_{7}$ | $(1,20,3,21,18,7,27)(2,13,24)(4,25,6,16)(5,11,19,14,22)(8,9,10)(12,23,17,15,26)$ |
| $C_{3} \cup 3 C_{4} \cup C_{5} \cup C_{7}$ | $(1,20,3,21,18,7,27)(2,24,11,5,13)(4,25,6,16)(8,9,10,22)(12,15,26)(14,19,17,23)$ |
| $5 C_{3} \cup C_{5} \cup C_{7}$ | $(1,20,3,19,10,7,27)(2,13,24)(4,16,25,8,23)(5,11,14)(6,26,12)(9,21,15)(17,18,22)$ |
| $5 C_{4} \cup C_{7}$ | $(1,27,7,18,19,3,20)(2,24,5,13)(4,15,26,12)(6,21,9,11)(8,16,17,23)(10,22,14,25)$ |
| $\begin{array}{r} 4 C_{3} \cup 2 C_{4} \cup C_{7} \cup C_{3} \cup 4 C_{6} \end{array}$ | $(1,20,3,16,9,7,27)(2,13,24)(4,18,6,23)(5,15,26,12)(8,10,25)(11,21,19)(14,17,22)$ <br> $(1,20,14,24,3,15)(2,27,12,5,10,21)(4,22,6,16,9,26)(7,13,19,18,23,17)(8,25,11)$ |
| $C_{4} \cup C_{5} \cup 3 C_{6}$ | $(1,15,3,24,14,20)(2,27,12,5,10,21)(4,22,6,16,9,26)(7,13,11,8,25)(17,19,18,23)$ |
| $3 C_{3} \cup 3 C_{6}$ | $(1,20,17,24,3,15)(2,27,12,23,10,21)(4,26,14,5,19,13)(6,16,22)(7,18,8)(9,25,11)$ |
| $3 C_{5} \cup 2 C_{6}$ | $(1,20,16,24,3,15)(2,27,12,8,11,21)(4,13,25,9,26)(5,10,23,14,17)(6,18,7,19,22)$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup 2 C_{6}$ | $(1,20,17,24,3,15)(2,21,19,12,27)(4,13,23,11,9,26)(5,10,14)(6,16,22)(7,18,8,25)$ |
| $C_{3} \cup 3 C_{4} \cup 2 C_{6}$ | $(1,15,3,24,8,20)(2,27,12,21)(4,11,9,26)(5,19,22,14,23,17)(6,25,13)(7,18,16,10)$ |
| $5 C_{3} \cup 2 C_{6}$ | $(1,20,15)(2,27,12,5,10,21)(3,17,24)(4,26,14)(6,16,22)(7,18,19)(8,11,13,23,9,25)$ |
| $2 C_{3} \cup 3 C_{5} \cup C_{6}$ | $(1,20,13,24,3,15)(2,21,7,12,27)(4,26,14,6,11)(5,19,17)(8,18,23)(9,16,22,10,25)$ |
| ${ }_{3} \cup 2 C_{4} \cup 2 C_{5} \cup C_{6}$ | $(1,20,17,24,3,15)(2,27,12,21)(4,16,9,26)(5,14,8,7,19)(6,25,13,23,11)(10,18,22)$ |
| $4 C_{4} \cup C_{5} \cup C_{6}$ | $(1,20,17,24,3,15)(2,27,12,21)(4,13,9,26)(5,19,6,14)(7,25,11,23,8)(10,18,22,16)$ |
| $4 C_{3} \cup C_{4} \cup C_{5} \cup C_{6}$ | $(1,15,3,24,14,20)(2,27,12,21)(4,26,11)(5,19,17)(6,13,7,18,22)(8,23,9)(10,16,25)$ |
| $3 C_{3} \cup 3 C_{4} \cup C_{6}$ | $(1,20,18,10,5,15)(2,27,12,21)(3,24,13,22)(4,26,14)(6,16,25)(7,19,17)(8,9,23,11)$ |
| $7 C_{3} \cup C_{6}$ | $(1,17,2,13,3,26)(4,19,20)(11,14,21)(9,12,24)(6,16,25)(5,15,23)(7,10,27)(8,18,22)$ |
| $C_{3} \cup C_{4} \cup 4 C_{5}$ | $(1,21,15,18,14)(2,16,25,5,24)(3,17,23,4,20)(6,22,12,19)(7,10,9)(8,27,11,26,13)$ |
| $3 C_{4} \cup 3 C_{5}$ | $(1,21,19,17,14)(2,16,25,5,18)(3,22,6,23)(4,15,12,20)(7,10,24,9)(8,27,11,26,13)$ |
| $4 C_{3} \cup 3 C_{5}$ | $(1,21,3,17,14)(2,16,25)(4,19,15)(5,23,9,24,12)(6,10,20)(7,18,22)(8,27,11,26,13)$ |
| $3 C_{3} \cup 2 C_{4} \cup 2 C_{5}$ | $(1,21,3,14)(2,16,24)(4,25,5,15)(6,19,17)(7,20,12,22,9)(8,27,11,26,13)(10,18,23)$ |
| $2 C_{3} \cup 4 C_{4} \cup C_{5}$ | $(1,21,19,14)(2,18,16)(3,22,4,20)(5,23,6,25)(7,10,9)(8,27,11,26,13)(12,15,17,24)$ |
| $6 C_{3} \cup C_{4} \cup C_{5}$ | $(1,21,20,3,14)(2,18,22)(4,24,12,25)(5,26,13)(6,19,15)(7,9,23)(8,27,11)(10,17,16)$ |
| $C_{3} \cup 6 C_{4}$ | $(1,18,22,17)(2,24,9,15)(3,25,12,26)(4,19,8,27)(5,10,6,16)(7,13,23)(11,21,20,14)$ |
| $5 C_{3} \cup 3 C_{4}$ | $(1,18,22,17)(2,19,15)(3,26,12)(4,24,8,27)(5,25,11)(6,21,20)(7,13,9,16)(10,14,23)$ |
| $9 C_{3}$ | $(1,15,26)(2,27,13)(3,16,23)(4,18,20)(5,25,12)(6,19,17)(7,21,14)(8,24,10)(9,22,11)$ |

Table 19: Strong VMTLs of the first 70 2-regular graphs of order 29.

|  |  |  |
| ---: | ---: | :--- | :--- |

Table 20: Strong VMTLs of the second 70 2-regular graphs of order 29.


Table 21: Strong VMTLs of the third 70 2-regular graphs of order 29.

|  | Ed |
| :---: | :---: |
| $3 C_{3} \cup C_{4} \cup C_{5} \cup C_{11}$ | $(1,20,19,12,25)(2,15,3,16,14,27,5,28,10,13,22)(4,18,24)(6,21,23)(7,29,11,9)(8,17,26)$ |
|  | $(1,25,14,21,6,16,3,15,2,22,20)(4,19,24)(5,27,10,28)(7,29,11,9)(8,23,18,26)(12,17,13)$ |
|  |  |
| $C_{9} \cup 2 C_{10}$ | ( $27,8,16,5,20,6,25,17)(2,21,23,4,18,12,7,22,19,15)(3,13,24,9,11,28,10,26,14,29)$ |
| $C_{3} \cup C_{6} \cup 2 C_{10}$ | $(1,17,25,12,8,27)(2,21,9,16,5,19,22,4,18,15)(3,13,6,23,11,28,10,26,14,29)(7,20,24)$ |
| $C_{4} \cup C_{5} \cup 2 C_{10}$ | $(1,27,8,25,17)(2,21,9,15)(3,29,14,26,10,28,11,20,24,13)(4,16,5,22,12,7,19,6,23,18)$ |
| $3 C_{3} \cup 2 C_{10}$ | $(1,17,7,12,21,2,19,23,8,27)(3,13,4,18,11,28,10,26,14,29)(5,22,15)(6,20,24)(9,16,25)$ |
| $C_{3} \cup C_{7} \cup C_{9} \cup C_{10}$ | $(1,27,8,18,4,16,9,25,17)(2,21,12,7,23,6,15)(3,29,14,26,10,28,11,20,24,13)(5,19,22)$ |
| $C_{4} \cup C_{6} \cup C_{9} \cup C_{1}$ | $(1,17,25,12,22,5,16,8,27)(2,21,23,18,4,15)(3,13,20,9,11,28,10,26,14,29)(6,24,7,19)$ |
| $2 C_{5} \cup C_{9} \cup C_{10}$ | $(1,27,8,25,17)(2,21,20,24,7,19,5,22,15)(3,13,6,23,11,28,10,26,14,29)(4,18,12,9,16)$ |
| ${ }_{3} \cup C_{4} \cup C_{9} \cup C_{10}$ | $(1,17,4,20,24,6,25,8,27)(2,15,5,21)(3,13,9,16,11,28,10,26,14,29)(7,12,22)(18,23,19)$ |
| $C_{3} \cup 2 C_{8} \cup C_{10}$ | $(1,27,8,23,18,4,16,5,25,17)(2,21,6,19,7,12,22,15)(3,13,11,28,10,26,14,29)(9,20,24)$ |
| $\cup C_{7} \cup C_{8} \cup C_{10}$ | $(1,27,8,22,19,25,17)(2,15,5,20,6,23,4,18,16,21)(3,13,11,28,10,26,14,29)(7,12,9,24)$ |
| $C_{5} \cup C_{6} \cup C_{8}$ | $(1,27,8,25,17)(2,21,9,16,5,15)(3,13,11,28,10,26,14,29)(4,23,6,20,24,7,12,22,19,18)$ |
| $2_{3} C_{3} \cup C_{5} \cup C_{8} \cup C_{1}$ | $(1,27,8,22,12,7,19,6,25,17)(2,21,16,5,15)(3,13,11,28,10,26,14,29)(4,23,18)(9,20,24)$ |
| $C_{3} \cup 2 C_{4} \cup C_{8} \cup C_{10}$ | $(1,27,8,17)(2,15,5,16,25,6,20,24,9,21)(3,13,11,28,10,26,14,29)(4,23,19,18)(7,12,22)$ |
| $C_{5} \cup 2 C_{7} \cup C_{10}$ | $(1,27,8,22,5,20,17)(2,15,9,12,21)(3,13,6,23,11,28,10,26,14,29)(4,18,24,7,19,25,16)$ |
| $2 C_{6} \cup C_{7} \cup C_{10}$ | $(1,17,25,5,16,8,27)(2,21,23,18,4,15)(3,13,24,9,11,28,10,26,14,29)(6,20,7,22,12,19)$ |
|  | $(1,17,25,6,16,8,27)(2,19,22,15,5,21)(3,13,4,23,11,28,10,26,14,29)(7,18,12)(9,20,24)$ |
| $\cup C_{4} \cup C_{5}$ | $(1,17,24,9,12,8,27)(2,15,16,21)(3,13,6,23,11,28,10,26,14,29)(4,22,20,7,18)(5,19,25)$ |
| $3 C_{4} \cup C_{7} \cup C_{1}$ | $(1,27,8,22,12,25,17)(2,21,23,18)(3,13,4,15,11,28,10,26,14,29)(5,19,6,16)(7,24,9,20)$ |
| $4 C_{3} \cup C_{7} \cup C_{10}$ | $(1,17,12,5,16,8,27)(2,20,21)(3,13,7,23,11,28,10,26,14,29)(4,15,22)(6,19,25)(9,18,24)$ |
| $C_{3} \cup C_{4} \cup 2 C_{6} \cup C_{10}$ | $(1,27,8,16,5,17)(2,21,4,15)(3,13,7,22,11,28,10,26,14,29)(6,20,24)(9,25,12,19,23,18)$ |
| $C_{3} \cup 2 C_{5} \cup C_{6} \cup C_{10}$ | $(1,27,8,25,17)(2,15,4,18,16,21)(3,13,12,9,11,28,10,26,14,29)(5,19,22)(6,20,24,7,23)$ |
| $2 C_{4} \cup C_{5} \cup C_{6} \cup C_{10}$ | $(1,27,8,25,17)(2,21,9,16,5,15)(3,29,14,26,10,28,11,20,24,13)(4,23,6,18)(7,19,22,12)$ |
| $3 C_{3} \cup C_{4} \cup C_{6} \cup C_{10}$ | $(1,17,7,12,8,27)(2,25,9,21)(3,13,4,22,11,28,10,26,14,29)(5,20,24)(6,16,15)(18,23,19)$ |
| $C_{4} \cup 3 C_{5} \cup C_{10}$ | $(1,27,8,25,17)(2,21,23,18,19)(3,13,4,16,11,28,10,26,14,29)(5,20,6,24)(7,15,9,22,12)$ |
| $3 C_{3} \cup 2 C_{5} \cup C_{10}$ | $(1,27,8,16,17)(2,20,21)(3,13,7,23,11,28,10,26,14,29)(4,15,22)(5,24,18,9,12)(6,19,25)$ |
| ${ }_{3} \cup 2 C_{4} \cup C_{5} \cup C_{10}$ | $(1,27,8,16,17)(2,15,5,21)(3,13,6,19,11,28,10,26,14,29)(4,18,23)(7,22,20,24)(9,25,12)$ |
| $C_{3} \cup 4 C_{4}$ | $(1,27,8,17)(2,24,9,21)(3,13,18,23,11,28,10,26,14,29)(4,15,6,16)(5,19,25,12)(7,22,20)$ |
|  | $(1,17,5,22,3,29,14,26,8,27)(2,15,21)(4,12,25,16)(6,13,18)(7,23,19)(9,20,24)(10,28,11)$ |
| $C_{3} \cup C_{8} \cup 2 C_{9}$ | $(1,22,16)(2,23,10,11,7,19,5,14)(3,17,18,4,26,13,21,6,25)(8,29,12,28,15,27,9,20,24)$ |
| $C_{4} \cup C_{7} \cup 2 C_{9}$ | $(1,22,10,19,7,14,2,23,16)(3,17,5,13,21,6,25)(4,20,18,26)(8,11,24,9,27,15,28,12,29)$ |
| $C_{5} \cup C_{6} \cup 2 C_{9}$ | $(1,22,5,19,16)(2,23,13,26,4,18,11,7,14)(3,17,21,10,9,25)(6,20,24,8,29,12,28,15,27)$ |
| $2 C_{3} \cup C_{5} \cup 2 C_{9}$ | $(1,22,16)(2,23,4,18,13,21,11,7,14)(3,17,27,15,28,12,29,8,25)(5,19,20,6,24)(9,26,10)$ |
|  | $(1,22,16)(2,23,13,19,25,3,17,5,14)(4,26,9,20)(6,21,18,8,29,12,28,15,27)(7,11,10,24)$ |
| $C_{4} \cup 2 C_{8} \cup C_{9}$ | $(1,22,11,10,9,18,20,6,16)(2,14,21,23)(3,25,4,26,13,5,19,17)(7,24,8,29,12,28,15,27)$ |
| $C_{5} \cup C_{7} \cup C_{8} \cup C_{9}$ | $(1,22,14,2,23,6,16)(3,17,4,26,13,5,19,25)(7,27,15,28,12,29,8,18,20)(9,24,11,21,10)$ |
| ${ }_{2} C_{6} \cup C_{8} \cup C_{9}$ | $(1,16,20,24,9,22)(2,23,6,18,4,26,13,5,14)(3,25,10,11,21,17)(7,27,15,28,12,29,8,19)$ |
| $2 C_{3} \cup C_{6} \cup C_{8} \cup C_{9}$ | $(1,22,16)(2,23,6,18,4,26,13,5,14)(3,17,19,7,20,24,9,25)(8,27,15,28,12,29)(10,21,11)$ |
| $\cup C_{4} \cup C_{5} \cup C_{8} \cup C_{9}$ | $(1,22,16)(2,23,11,24,9,20,7,14)(3,17,19,25)(4,26,13,6,18)(5,21,10,8,29,12,28,15,27)$ |
| $3 C_{4}$ | $(1,22,13,21,5,14,2,23,16)(3,17,19,25)(4,26,6,18)(7,24,9,20)(8,10,11,27,15,28,12,29)$ |
| $4 C_{3} \cup C_{8} \cup C_{9}$ | $(1,22,16)(2,23,13,26,5,17,3,18,14)(4,20,24)(6,21,8,29,12,28,15,27)(7,19,11)(9,25,10)$ |
| $C_{6} \cup 2 C_{7} \cup C_{9}$ | $(1,22,4,26,13,6,16)(2,14,18,20,7,11,10,21,23)(3,17,19,5,24,9,25)(8,29,12,28,15,27)$ |
| $2 C_{3} \cup 2 C_{7} \cup C_{9}$ | $(1,22,16)(2,23,10,9,26,4,14)(3,25,11,20,24,5,17)(6,21,18)(7,19,13,8,29,12,28,15,27)$ |
| $\cup C_{4} \cup C_{6} \cup C_{7} \cup C_{9}$ | $(1,22,16)(2,14,25,3,17,19,5,21,23)(4,26,6,13,20,9,18)(7,11,10,24)(8,29,12,28,15,27)$ |
| $C_{3} \cup 2 C_{5} \cup C_{7}$ | $(1,22,16)(2,23,11,7,14)(3,25,10,9,13,19,17)(4,20,24,5,26)(6,21,18,8,29,12,28,15,27)$ |
|  | $(1,16,5,13,21,23,2,14,22)(3,25,10,17)(4,26,6,18)(7,24,9,20,19)(8,29,12,28,15,27,11)$ |
| $3 C_{3} \cup C_{4} \cup C_{7} \cup C_{9}$ | $(1,22,16)(2,23,9,18,11,7,14)(3,19,17)(4,20,24)(5,25,10,21)(6,13,26,8,29,12,28,15,27)$ |
| $C_{3} \cup C_{5} \cup 2 C_{6} \cup C_{9}$ | $(1,22,16)(2,14,5,21,23)(3,25,6,18,4,26,13,19,17)(7,11,10,24,9,20)(8,29,12,28,15,27)$ |
| $2 C_{4} \cup 2 C_{6} \cup C_{9}$ | $(1,22,5,16)(2,14,21,23)(3,17,19,10,9,25)(4,26,13,20,6,18)(7,24,8,29,12,28,15,27,11)$ |
| $C_{4} \cup 2 C_{5} \cup C_{6} \cup C_{9}$ | $(1,22,9,20,7,11,21,5,16)(2,23,10,24,14)(3,17,19,25)(4,26,13,6,18)(8,29,12,28,15,27)$ |
| $3 C_{3} \cup C_{5} \cup C_{6} \cup C_{9}$ | $(1,22,16)(2,23,11,25,14)(3,17,4,24,5,13,20,7,19)(6,18,26)(8,29,12,28,15,27)(9,10,21)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{6} \cup C_{9}$ | $(1,22,16)(2,23,4,26,6,18,21,5,14)(3,17,19,25)(7,11,10,24)(8,29,12,28,15,27)(9,20,13)$ |
| $4 C_{5} \cup C_{9}$ | $(1,16,5,13,22)(2,23,10,24,14)(3,17,7,19,25)(4,26,6,21,18)(8,29,12,28,15,27,9,20,11)$ |
| $2 C_{3} \cup C_{4} \cup 2 C_{5} \cup C_{9}$ | $(1,22,16)(2,23,11,7,14)(3,17,27,15,28,12,29,8,25)(4,20,19,13,18)(5,21,6,24)(9,26,10)$ |
| $C_{3} \cup 3 C_{4} \cup C_{5} \cup C_{9}$ | $(1,22,16)(2,23,13,5,14)(3,17,27,15,28,12,29,8,19)(4,20,6,24)(7,25,9,26)(10,21,18,11)$ |
| $5 C_{3} \cup C_{5}$ | $(1,22,5,23,16)(2,20,14)(3,26,10,8,29,12,28,15,27)(4,21,17)(6,13,18)(7,19,25)(9,24,11)$ |
|  | $(1,22,17,16)(2,14,21,23)(3,26,6,24)(4,18,20,8,29,12,28,15,27)(5,13,7,19)(9,10,11,25)$ |
| $4 C_{3} \cup 2 C_{4} \cup C_{9}$ | $(1,16,22)(2,23,6,14)(3,18,4,24)(5,13,21)(7,17,27,15,28,12,29,8,25)(9,26,10)(11,20,19)$ |
| $C_{5} \cup 3 C_{8}$ | $(1,18,6,25,8,21,4,26)(2,19,3,20,22,10,29,14)(5,15,23,11,7,28,16,12)(9,27,13,24,17)$ |
| $C_{6} \cup C_{7} \cup 2 C_{8}$ | $(1,18,8,24,4,26)(2,19,3,20,21,10,29,14)(5,15,22,11,23,6,12)(7,28,16,9,27,13,25,17)$ |
| $2 C_{3} \cup C_{7} \cup 2 C_{8}$ | $(1,18,6,11,7,28,16,26)(2,19,3,20,10,29,14)(4,22,9,27,13,21,12,25)(5,23,15)(8,17,24)$ |
| $C_{3} \cup C_{4} \cup C_{6} \cup 2 C_{8}$ | $(1,18,11,6,12,22,4,26)(2,19,3,20,21,10,29,14)(5,15,23)(7,28,16,8,17,25)(9,27,13,24)$ |
| $\mathrm{C}_{3} \cup 2 \mathrm{C}_{5} \cup 2 \mathrm{C}_{8}$ | $(1,26,12,5,18)(2,19,3,15,22,10,29,14)(4,24,7,28,16)(6,23,11,13,27,9,21,20)(8,17,25)$ |
| $2 C_{4} \cup C_{5} \cup 2 C_{8}$ | $(1,26,11,6,18)(2,19,3,20,8,10,29,14)(4,22,12,21)(5,25,17,24)(7,28,16,15,23,9,27,13)$ |
| $\begin{aligned} & 3 C_{3} \cup C_{4} \cup 2 C_{8} \\ & 3 C_{7} \cup C_{8} \end{aligned}$ | $(1,26,12,18)(2,19,3,23,8,10,29,14)(4,21,20)(5,15,17,25,9,27,13,24)(6,22,11)(7,28,16)$ <br> $(1,18,6,23,11,15,26)(2,19,3,20,22,10,29,14)(4,24,7,28,16,17,21)(5,25,12,8,9,27,13)$ |
| $C_{3} \cup C_{4} \cup 2 C_{7} \cup C_{8}$ | $(1,18,24,13,27,9,8,26)(2,19,3,20,10,29,14)(4,22,11,21)(5,23,15)(6,12,17,7,28,16,25)$ |
| $C_{3} \cup C_{5} \cup C_{6} \cup C_{7} \cup C_{8}$ | $(1,18,8,21,4,26)(2,19,3,20,22,10,29,14)(5,23,15)(6,12,25,9,27,13,11)(7,28,16,17,24)$ |

Table 22: Strong VMTLs of the remaining 63 2-regular graphs of order 29.

| Graph | Edge Labels |
| :---: | :---: |
| $2_{2} C_{4} \cup C_{6} \cup C_{7} \cup C_{8}$ | $(1,18,7,28,16,4,26)(2,19,3,20,22,10,29,14)(5,21,17,24)(6,12,25,8,23,11)(9,27,13,15)$ |
| $C_{4} \cup 2 C_{5} \cup C_{7} \cup C_{8}$ | $(1,26,6,18)(2,19,3,20,8,10,29,14)(4,22,11,9,27,13,21)(5,24,17,25,12)(7,28,16,15,23)$ |
| $3 C_{3} \cup C_{5} \cup C_{7} \cup C_{8}$ | $(1,26,11,22,8,17,24,18)(2,19,3,21,10,29,14)(4,25,9,27,13)(5,15,23)(6,12,20)(7,28,16)$ |
| $2 C_{3} \cup 2 C_{4} \cup C_{7} \cup C_{8}$ | $(1,26,7,28,16,21,11,18)(2,19,3,15,10,29,14)(4,22,20)(5,23,8,12)(6,17,24)(9,27,13,25)$ |
| $C_{3} \cup 3 C_{6} \cup C_{8}$ | $(1,18,5,24,6,26)(2,19,3,17,8,10,29,14)(4,20,22,9,27,13)(7,28,16,25,12,21)(11,15,23)$ |
| $C_{4} \cup C_{5} \cup 2 C_{6} \cup C_{8}$ | $(1,18,24,8,26)(2,19,3,20,21,10,29,14)(4,22,15,23,5,25)(6,11,9,27,13,12)(7,28,16,17)$ |
| $3 C_{3} \cup 2 C_{6} \cup C_{8}$ | $(1,18,22,20,12,26)(2,19,3,25,8,10,29,14)(4,21,5,15,9,27)(6,23,11)(7,28,16)(13,17,24)$ |
| $3 C_{5} \cup C_{6} \cup C_{8}$ | $(1,26,8,12,5,18)(2,19,3,23,15,10,29,14)(4,20,22,6,25)(7,28,16,21,11)(9,27,13,17,24)$ |
| $2 C_{3} \cup C_{4} \cup C_{5} \cup C_{6} \cup C_{8}$ | $(1,26,6,24,18)(2,19,3,23,8,10,29,14)(4,21,20)(5,15,22,11,17,12)(7,28,16)(9,27,13,25)$ |
| $C_{3} \cup 3 C_{4} \cup C_{6} \cup C_{8}$ | $(1,26,11,18)(2,19,3,17,24,10,29,14)(4,20,22,6,12,13)(5,21,9,27)(7,28,16)(8,23,15,25)$ |
| $5 C_{3} \cup C_{6} \cup C_{8}$ | $(1,18,22,8,9,27,6,26)(2,19,12,10,29,14)(3,17,25)(4,21,20)(5,24,13)(7,28,16)(11,15,23)$ |
| $2 C_{3} \cup 3 C_{5} \cup C_{8}$ | $(1,26,12,13,18)(2,19,3,21,8,10,29,14)(4,22,20,17,24)(5,15,25)(6,11,23,9,27)(7,28,16)$ |
| $C_{3} \cup 2 C_{4} \cup 2 C_{5} \cup C_{8}$ | $(1,18,24,4,26)(2,19,3,15,22,10,29,14)(5,21,20)(6,23,8,12,11)(7,28,16,17)(9,27,13,25)$ |
| $4 C_{4} \cup C_{5} \cup C_{8}$ | $(1,18,12,26)(2,19,3,25,8,10,29,14)(4,21,20,22)(5,15,9,27)(6,17,23,11)(7,28,16,13,24)$ |
| $4 C_{3} \cup C_{4} \cup C_{5} \cup C_{8}$ | $(1,26,4,13,18)(2,19,21,3,15,10,29,14)(5,24,17)(6,22,20)(7,28,16)(8,25,12)(9,27,11,23)$ |
| $3 C_{3} \cup 3 C_{4} \cup C_{8}$ | $(1,26,8,18)(2,19,12,6,23,10,29,14)(3,21,20,22)(4,13,24)(5,17,15,25)(7,28,16)(9,27,11)$ |
| $7 C_{3} \cup C_{8}$ | $(1,20,21)(9,15,29)(5,11,28)(2,17,26)(8,18,24)(3,14,22)(6,12,25)(7,23,4,16,19,10,13,27)$ |
| $C_{3} \cup C_{5} \cup 3 C_{7}$ | $(1,19,24,2,22,5,20)(3,29,15,27,9,14,26)(4,13,18)(6,28,11,8,10)(7,21,17,16,25,12,23)$ |
| $2 C_{4} \cup 3 C_{7}$ | $(1,19,8,23,5,13,20)(2,22,21,17)(3,29,15,27,9,14,26)(4,12,25,16,10,7,18)(6,28,11,24)$ |
| $C_{3} \cup 2 C_{6} \cup 2 C_{7}$ | $(1,20,13,17,24,19)(2,22,16)(3,29,15,27,9,14,26)(4,12,25,10,7,18)(5,21,6,28,11,8,23)$ |
| $\cup C_{5} \cup C_{6} \cup 2 C_{7}$ | $(1,20,2,22,8,19)(3,29,15,27,9,14,26)(4,24,7,10,16,17,21)(5,11,28,6,13)(12,23,18,25)$ |
| $3 C_{3} \cup C_{6} \cup 2 C_{7}$ | $(1,20,2,22,8,19)(3,29,15,27,9,17,26)(4,14,21)(5,11,28,6,13,25,12)(7,16,24)(10,18,23)$ |
| $3 C_{5} \cup 2 C_{7}$ | $(1,20,23,2,22,16,19)(3,29,15,27,9,14,26)(4,24,17,13,18)(5,11,28,6,21)(7,12,25,8,10)$ |
| $C_{3} \cup C_{4} \cup C_{5} \cup 2 C_{7}$ | $(1,19,22,2,17,8,20)(3,29,15,27,9,7,26)(4,18,25,12,23)(5,13,10,21)(6,28,11)(14,16,24)$ |
| $C_{3} \cup 3 C_{4} \cup 2 C_{7}$ | $(1,20,8,19)(2,22,16,24)(3,29,15,27,9,7,26)(4,14,5,17,13,10,21)(6,11,28)(12,23,18,25)$ |
| $5 C_{3} \cup 2 C_{7}$ | $(9,19,23)(2,15,24)(11,16,25)(5,18,26)(6,13,27)(1,17,3,22,21,8,29)(7,14,20,4,12,10,28)$ |
| $C_{4} \cup 3 C_{6} \cup C_{7}$ | $(1,19,8,22,2,20)(3,29,15,27,9,26)(4,12,25,18,23,5,21)(6,28,11,7,10,13)(14,17,16,24)$ |
| $2 C_{5} \cup 2 C_{6} \cup C_{7}$ | $(1,19,24,7,20)(2,23,18,4,13,22)(3,29,15,27,9,14,26)(5,25,12,16,17,21)(6,28,11,8,10)$ |
| $2 C_{3} \cup C_{4} \cup 2 C_{6} \cup C_{7}$ | $(1,19,7,10,8,20)(2,21,22)(3,29,15,27,9,26)(4,18,23)(5,11,28,6,13,12,25)(14,17,16,24)$ |
| $2 C_{3} \cup 2 C_{5} \cup C_{6} \cup C_{7}$ | $(1,19,24,17,8,20)(2,22,16)(3,29,15,27,9,14,26)(4,13,18)(5,11,28,6,21)(7,23,10,25,12)$ |
| $3 \cup 2 C_{4} \cup C_{5} \cup C_{6} \cup C_{7}$ | $(1,20,2,22,16,19)(3,29,15,27,9,14,26)(4,13,17,24)(5,11,28,6,21)(7,18,25,12)(8,23,10)$ |
| $4 C_{4} \cup C_{6} \cup C_{7}$ | $(1,19,4,20)(2,17,24,14)(3,29,15,27,9,26)(5,25,12,13)(6,28,11,16)(7,21,22,18,8,23,10)$ |
| $4 C_{3} \cup C_{4} \cup C_{6} \cup C_{7}$ | $(1,19,4,13,18,20)(2,17,23)(3,29,15,27,9,7,26)(5,21,22)(6,28,11,24)(8,14,10)(12,16,25)$ |
| $C_{3} \cup C_{4} \cup 3 C_{5} \cup C_{7}$ | $(1,19,22,2,20)(3,29,15,27,9,7,26)(4,23,5,13)(6,17,8,11,28)(10,16,24,14,21)(12,18,25)$ |
| $3 C_{4} \cup 2 C_{5} \cup C_{7}$ | $(1,20,8,19)(2,23,17,21,22)(3,29,15,27,9,7,26)(4,13,18,5,14)(6,24,11,28)(10,16,25,12)$ |
| $4 C_{3} \cup 2 C_{5} \cup C_{7}$ | $(1,19,22,4,20)(2,21,17)(3,29,15,27,9,7,26)(5,13,12)(6,28,11,16,24)(8,14,23)(10,18,25)$ |
| $C_{3} \cup 2 C_{4} \cup C_{5} \cup C_{7}$ | $(1,19,24,4,20)(2,16,10,21)(3,29,15,27,9,7,26)(5,14,8,22)(6,28,11)(12,25,13)(17,18,23)$ |
|  | $(1,19,21,20)(2,22,16,14)(3,29,15,27,9,17,26)(4,18,5,23)(6,28,11)(7,12,13,24)(8,25,10)$ |
| $6 C_{3} \cup C_{4} \cup C_{7}$ | $(1,17,3,22,21,8,29)(4,12,11,27)(6,13,28)(2,15,20)(9,18,24)(7,19,25)(10,14,26)(5,16,23)$ |
| $C_{5} \cup 4 C_{6}$ | $(1,22,7,11,24)(2,15,28,4,12,25)(3,23,10,20,21,18)(5,14,6,16,8,26)(9,19,17,27,13,29)$ |
| $2 C_{3} \cup C_{5} \cup 3 C_{6}$ | $(1,22,11,26,5,24)(2,25,16,4,28,15)(3,21,18)(6,20,14,8,10)(7,23,12)(9,19,17,27,13,29)$ |
| $C_{3} \cup 2 C_{4} \cup 3 C_{6}$ | $(1,22,11,24)(2,25,16,4,28,15)(3,18,12,6,20,19)(5,14,23)(7,17,27,13,29,9)(8,26,10,21)$ |
| $C_{3} \cup C_{4} \cup 2 C_{5} \cup 2 C_{6}$ | $(1,22,8,23,5,24)(2,15,28,4,12,25)(3,19,16)(6,18,21,20,14)(7,11,10,26)(9,17,27,13,29)$ |
| $3 C_{4} \cup C_{5} \cup 2 C_{6}$ | $(1,22,11,24)(2,15,28,4,25)(3,18,6,16)(5,13,29,9,7,23)(8,26,14,12)(10,21,20,19,17,27)$ |
| $4 C_{3} \cup C_{5} \cup 2 C_{6}$ | $(1,24,6,20,19,22)(2,25,4,28,15)(3,21,16)(5,11,23)(7,26,14)(8,12,10)(9,29,13,18,17,27)$ |
| $3 C_{3} \cup 2 C_{4} \cup 2 C_{6}$ | $(1,22,6,24)(2,15,28,4,12,25)(3,18,23)(5,17,27,7,26,14)(8,10,21)(9,29,13,11)(16,20,19)$ |
| $C_{3} \cup 4 C_{5} \cup C_{6}$ | $(1,24,10,8,22)(2,25,4,28,15)(3,18,6,14,19)(5,11,26)(7,21,20,16,23,12)(9,17,27,13,29)$ |
| $2 C_{4} \cup 3 C_{5} \cup C_{6}$ | $(1,22,11,7,24)(2,15,28,4,12,25)(3,19,5,16)(6,14,21,18,23)(8,20,10,26)(9,17,27,13,29)$ |
| $3 C_{3} \cup C_{4} \cup 2 C_{5} \cup C_{6}$ | $(1,22,9,29,13,24)(2,25,4,28,15)(3,16,19,17,27)(5,11,23)(6,18,21,20)(7,26,14)(8,12,10)$ |
| $2 C_{3} \cup 3 C_{4} \cup C_{5} \cup C_{6}$ | $(1,22,11,24)(2,25,4,28,15)(3,23,5,16)(6,18,19,17,27,12)(7,13,29,9)(8,26,14)(10,21,20)$ |
| $6 C_{3} \cup C_{5} \cup C_{6}$ | $(1,22,11,23,7,24)(2,19,25)(3,26,14)(4,28,15)(5,17,9,29,13)(6,18,10)(8,27,12)(16,21,20)$ |
| $C_{3} \cup 5 C_{4} \cup C_{6}$ | $(1,22,6,24)(2,25,16,4,28,15)(3,21,18)(5,11,26,14)(7,19,17,27)(8,23,12,10)(9,29,13,20)$ |
| $5 C_{3} \cup 2 C_{4} \cup C_{6}$ | $(1,24,12,22)(2,25,16,4,28,15)(3,21,18)(5,26,14)(6,23,10)(7,19,11)(8,20,17,27)(9,29,13)$ |
| $C_{4} \cup 5 C_{5}$ | $(1,24,18,10,19)(2,28,16,6,25)(3,20,23,13,21)(4,22,15,17)(5,11,7,26,14)(8,27,12,29,9)$ |
| $3 C_{3} \cup 4 C_{5}$ | $(1,19,24)(2,28,16,7,25)(3,18,13,21,15)(4,20,22)(5,14,26,11,17)(6,23,10)(8,9,29,12,27)$ |
| $2 C_{3} \cup 2 C_{4} \cup 3 C_{5}$ | $(1,24,10,19)(2,28,16,6,25)(3,13,15,21)(4,22,20,23,17)(5,14,18)(7,11,26)(8,27,12,29,9)$ |
| $C_{3} \cup 4 C_{4} \cup 2 C_{5}$ | $(1,19,18,24)(2,28,16,3,25)(4,20,6,17)(5,11,23,13)(7,26,14,15)(8,9,29,12,27)(10,21,22)$ |
| $5 C_{3} \cup C_{4} \cup 2 C_{5}$ | $(1,19,24)(2,28,16,7,25)(3,13,26,5,21)(4,15,18)(6,11,17,23)(8,27,10)(9,29,12)(14,20,22)$ |
| $6 C_{4} \cup C_{5}$ | $(1,19,4,24)(2,28,16,6,25)(3,14,15,18)(5,13,11,21)(7,12,29,9)(8,26,10,27)(17,22,20,23)$ |
| $4 C_{3} \cup 3 C_{4} \cup C_{5}$ | $(1,19,24)(2,28,16,3,25)(4,18,11,20)(5,13,21)(6,26,7,10)(8,27,15)(9,29,12)(14,22,17,23)$ |
| $8 C_{3} \cup C_{5}$ | $(1,19,24)(2,25,14)(3,21,15)(4,28,16,13,27)(5,17,18)(6,20,22)(7,23,10)(8,11,26)(9,29,12)$ |
| $3 C_{3} \cup 5 C_{4}$ | $(1,19,23,18)(2,22,3,25)(4,27,13)(5,29,14,16)(6,26,12,10)(7,11,28)(8,15,21)(9,17,20,24)$ |
| $7 C_{3} \cup 2 C_{4}$ | $(1,21,9,18)(2,23,15)(3,17,25)(4,19,22)(5,29,14,26)(6,27,10)(7,28,11)(8,16,13)(12,20,24)$ |

## References

[1] Wieb Bosma, John Cannon and Catherine Playoust, The Magma algebra system. I. The user language, J. Symbolic Comput. 24 (1997), 235-265. (http://magma.maths.usyd.edu.au/)
[2] I. D. Gray, Vertex-magic total labelings of regular graphs, SIAM J. Discrete Math. 21 (2007), no. 1, 170-177.
[3] I. D. Gray and J. A. MacDougall, Vertex-magic total labelings of regular graphs II, Discrete Math. 309 (2009), 5986-5999.
[4] J. A. MacDougall, Vertex-magic Labeling of Regular Graphs, lecture, July 2002, DIMACS Connect Institute.
[5] J. A. MacDougall, Mirka Miller, Slamin and W.D. Wallis, Vertex-magic Total Labelings of Graphs, Utilitas Math. 61 (2002), 3-21.
[6] D. McQuillan, Vertex-magic Cubic Graphs, J. Combin. Math. Combin. Comput. 48 (2004), 103-106.
[7] D. McQuillan, A technique for constructing magic labelings of 2-regular graphs, J. Combin. Math. Combin. Comput. 75 (2010), 129-135.
[8] M. Meringer, Fast Generation of Regular Graphs and Construction of Cages, J. Graph Theory 30 (1999), 137-146.
[9] N. J. A. Sloane, The On-Line Encyclopedia of Integer Sequences, published electronically at http://oeis.org (2010).

