ADDENDUM TO : MINIMUM WEIGHT SPANNING TREES WITH BOUNDED DIAMETER

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In an earlier paper [1] we presented a Mixed Integer Linear Programming (MILP) formulation for the following problem :

Find, in a given edge-weighted graph G, a minimum weight spanning tree of diameter at most D.

The formulation presented fails when D is odd and the optimal tree has diameter exactly D. The only error is in the procedure presented on pages 271 and 272. The objective of this note is to correct this error.

The simplest way to overcome the problem is to change slightly the formulation. Recall that in [1] we extended the given graph G to a directed graph G* with one source (s) and two sinks (t_1 and t_2). Further, our formulation restricted the out degree of the source s to one. Here we modify the directed graph by allowing only one sink (t) and restricting the out degree of the source s to at most $1 + \lceil \frac{1}{2} D \rceil - \lfloor \frac{1}{2} D \rfloor$; we call the resulting directed graph G* and use the notation in [1].

The correct MILP formulation is :

$$\text{Minimize } Z = \sum_{\substack{i \neq j \\ (i,j) \in E}} w^*(i,j) z_{ij} \tag{1}$$

subject to the constraints :

$$x_{ij} = 0 \text{ or } 1, \text{ for } i \neq j \text{ and } (i,j) \in E^* \tag{2}$$

$$1 \leq \sum_{i \in V} x_{si} \leq 1 + \lceil \tfrac{1}{2} D \rceil - \lfloor \tfrac{1}{2} D \rfloor \tag{3}$$

$$x_{si} + x_{sj} \leq 1, \text{ for all } (i,j) \notin E^* \tag{4}$$

$$x_{si} + \sum_{\substack{k \in V \\ k \neq i}} x_{ki} = 1, \quad \text{for all } i \in V \tag{5}$$

$$\sum_{\substack{k\in V\\k\neq i}} x_{ik} + x_{it} \geq 1, \text{ for all } i\in V \tag{6}$$

$$y_i - y_j + (L+1)x_{ij} \leq L, \text{ for all } i\neq j, (i,j)\in E^* \tag{7}$$

$$y_t - y_s \leq \lfloor \tfrac{1}{2}D \rfloor + 2 \tag{8}$$

$$x_{si} + x_{sj} - z_{ij} \leq 1 \text{ , for all } (i,j)\in E \tag{9}$$

$$z_{ij} \geq x_{ij} \text{ , for all } (i,j)\in E \tag{10}$$

Constraints (2) - (4) restrict the outdegree of s to be exactly one when D is even and at most two when D is odd. Constraints (5) - (7) play the same role as in [1], namely restricting the indegree (outdegree) of each vertex of G^* - s - t to one (at least one) and eliminate subtours. Given a feasible solution (x_{ij}, y_i) to constraints (2) - (8), observe that the corresponding z_{ij}'s will, by (1), (9) and (10), be forced to be

$$z_{ij} = \max\{x_{ij}, x_{si} + x_{sj} - 1\}.$$

Consequently, z_{ij}'s will be either 0 or 1. From this solution we identify a spanning tree T′ as follows.

$$T' = \{(i,j) : z_{ij} = 1 \text{ and } (i,j)\in E\}.$$

Using arguments similar to those used in proving Theorem 2.2 in [1] one can establish that $d(T') \leq D$.

Given any spanning tree T of G with $d(T) \leq D$, we construct a feasible solution (x_{ij}, z_{ij}, y_i) to the MILP problem (1) - (10) as follows :

Step 1 : Set $x_{ij} = 0$ for every $(i,j)\in E$.

Step 2 : Find the eccenticity $e(j)$ for every $j\in V$.

Step 3 : Find $I^* = \{i^* : e(i^*) = \min_{j\in V} e(j)\}$.

Step 4 : Set $x_{si^*} = 1$ for every $i^*\in I^*$, $x_{sj} = 0$ for every $j\notin I^*$, $y_s = 0$, $y_{i^*} = 1$ for every $i^*\in I^*$.
The vertices in I^* are said to be labelled.

Step 5 : Choose a labelled vertex i (this means that y_i is fixed) and carry out the following steps :

(i) If $(i,j)\in T$ and j is not yet labelled, then set $x_{ij} = 1$ and $y_j = y_i + 1$. Vertex j is now labelled.

(ii) If there does not exist any j such that $(i,j)\in T$ and j is not labelled, then set $x_{it} = 1$.

Step 6 : Repeat Step 5 until all vertices of G are labelled.

Step 7 : Set $y_t = \max_{i\in V}\{y_i\} + 1$ and $z_{ij} = \max\{x_{ij}, x_{si} + x_{sj} - 1\}$, for all i,j.

The above proceedure corrects the error we mentioned earlier.

[1] N.R. Achuthan and L. Caccetta, Minimum Weight Spanning Trees with Bounded Diameter, Australasian Journal of Combinatorics 5(1992), pp. 261-276.

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