Some new weighing matrices using sequences with zero autocorrelation function

Christos Koukouvinos
Department of Statistics
Athens University of Economics
76 Patission St, 104 34 Athens, Greece

and

Jennifer Seberry
Department of Computer Science
University of Woolongong
Woolongong, NSW 2522, Australia

Dedicated to the memory of Alan Rahilly, 1947 – 1992

Abstract

We verify the skew weighing matrix conjecture for orders $2^t.13$, $t \geq 5$, and give new results for $2^t.15$ proving the conjecture for $t \geq 3$.

1 Introduction

An *orthogonal design* A, of order n, and type $(s_1, s_2, \ldots, s_u)$, denoted $OD(n; s_1, s_2, \ldots, s_u)$ on the commuting variables $(\pm x_1, \pm x_2, \ldots, \pm x_u, 0)$ is a square matrix of order n with entries $\pm x_k$ where each x_k occurs s_k times in each row and column such that the distinct rows are pairwise orthogonal.

In other words

$$AA^T = (s_1 x_1^2 + \ldots + s_u x_u^2) I_n$$

where I_n is the identity matrix. It is known that the maximum number of variables in an orthogonal design is $\rho(n)$, the Radon number, where for $n = 2^a b$, b odd, set $a = 4c + d, 0 \leq d < 4$, then $\rho(n) = 8c + 2^d$.

A weighing matrix $W = W(n, k)$ is a square matrix with entries 0, ± 1 having k non-zero entries per row and column and inner product of distinct rows zero. Hence W satisfies $WW^T = kI_n$, and W is equivalent to an othogonal design $OD(n; k)$. The number k is called the *weight* of W.

Weighing matrices have long been studied because of their use in weighing experiments as first studied by Hotelling [8] and later by Raghavarao [9] and others.

There are a number of conjectures concerning weighing matrices:

Conjecture 1 (Wallis [13]) *There exists a weighing matrix* $W(4t, k)$ *for* $k \in \{1, \ldots, 4t\}$.

This conjecture was proved true for orders $n = 2^t$, t a positive integer by Geramita, Pullman and (Seberry) Wallis [3]. Later the conjecture was made stronger by Seberry until it appeared in the following forms.

Conjecture 2 (Seberry) *When* $n \equiv 4 (mod\ 8)$, *there exist a skew-weighing matrix (also written as an* $OD(n; 1, k)$*) when* $k \le n-1$, $k = a^2 + b^2 + c^2$, a, b, c *integers except that* $n-2$ *must be the sum of two squares.*

Conjecture 3 (Seberry) *When* $n \equiv 0 (mod\ 8)$, *there exist a skew-weighing matrix (also written as an* $OD(n; 1, k)$*) for all* $k \le n-1$.

This conjecture was established for $n = 2^t.3$, $2^t.5$, $2^t.9$ by Geramita and (Seberry) Wallis [4, 5], by Eades and (Seberry) Wallis [1] for $t \ge 3$ and for $n = 2^t.15$ and $2^t.21$, $t \ge 4$ by Seberry [10, 11]. The result for $2^t.15$ is improved to $t \ge 3$ in this paper and the results are given for $2^t.13$, for $t \ge 5$.

Given the sequence $A = \{a_1, a_2, ..., a_n\}$ of length n the *non-periodic autocorrelation function* $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \qquad s = 0, 1, ..., n-1. \tag{1}$$

If $A(z) = a_1 + a_2 z + \cdots + a_n z^{n-1}$ is the associated polynomial of the sequence A, then

$$A(z)A(z^{-1}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}),\ z \neq 0. \tag{2}$$

Given A as above of length n the *periodic autocorrelation function* $P_A(s)$ is defined, reducing $i + s$ modulo n, as

$$P_A(s) = \sum_{i=1}^{n} a_i a_{i+s}, \qquad s = 0, 1, ..., n-1. \tag{3}$$

2 Preliminary Results

We make extensive use of the book of Geramita and Seberry [6]. We quote the following theorems, giving their reference from the aforementioned book, that we use:

Lemma 1 **[6, Lemma 4.11]** *If there exists an orthogonal design* $OD(n; s_1, s_2, \ldots, s_u)$ *then there exists an orthogonal design* $OD(2n; s_1, s_1, es_2, \ldots, es_u)$ *where* $e = 1$ *or* 2.

Lemma 2 **[6, Lemma 4.4]** *If* A *is an orthogonal design* $OD(n; s_1, s_2, \ldots, s_u)$ *on the commuting variables* $(\pm x_1, \pm x_2, \ldots, \pm x_u, 0)$ *then there is an orthogonal design* $OD(n; s_1, s_2, \ldots, s_i + s_j, \ldots, s_u)$ *and* $OD(n; s_1, s_2, \ldots, s_{j-1}, s_{j+1}, \ldots, s_u)$ *on the* $u-1$ *commuting variables* $(\pm x_1, \pm x_2, \ldots, \pm x_{j-1}, \pm x_{j+1}, \ldots, \pm x_u, 0)$.

Lemma 3 **[6, Corollary 5.2]** *If all orthogonal designs* $OD(n; 1, k)$, $k = 1, 2, \cdots, n-1$, *exist then all orthogonal design* $OD(2n; 1, j)$, $j = 1, 2, \cdots, 2n-1$, *exist.*

Theorem 1 **[6, Theorems 2.19 and 2.20]** *Suppose* $n \equiv 0 (mod\ 4)$. *Then the existence of a* $W(n, n-1)$ *implies the existence of a skew-symmetric* $W(n, n-1)$. *The existence of a skew-symmetric* $W(n, k)$ *is equivalent to the existence of an* $OD(n; 1, k)$.

Theorem 2 **[6, Proposition 3.54 and Theorem 2.20]** *An orthogonal design* $OD(n; 1, k)$ *can only exist in order* $n \equiv 4 (mod\ 8)$ *if* k *is the sum of three squares. An orthogonal design* $OD(n; 1, n-2)$ *can only exist in order* $n \equiv 4 (mod\ 8)$ *if* $n-2$ *is the sum of two squares.*

Theorem 3 *Orthogonal designs* $OD(n; 1, k)$ *exist for* $k = 1, 2, \cdots, n-1$ *in orders* $n = 2^t, 2^{t+3}.3, 2^{t+3}.5, 2^{t+3}.7, 2^{t+3}.9, 2^{t+4}.15$ *and* $2^{t+4}.21$, $t \geq 0$ *an integer.*

Theorem 4 **[6, Theorem 4.49]** *If there exist four circulant matrices* A_1, A_2, A_3, A_4 *of order* n *satisfying*

$$\sum_{i=1}^{4} A_i A_i^T = fI$$

where f *is the quadratic form* $\sum_{j=1}^{u} s_j x_j^2$, *then there is an orthogonal design* $OD(n; s_1, s_2, \ldots, s_u)$.

Corollary 1 *If there are four* $\{0, \pm 1\}$*-sequences of length* n *and weight* w *with zero periodic or non-periodic autocorrolation function then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals-Seidel array to form* $OD(4n; w)$ *or a* $W(4n, w)$. *If one of the sequences is skew-type then they can be used similarly to make an* $OD(4n; 1, w)$. *We note that if there are sequences of length* n *with zero non-periodic autocorrelation function then there are sequences of length* $n + m$ *for all* $m \geq 0$.

Theorem 5 **[6, Theorems 4.124 and 4.41]** *Let* q *be a prime power then there is a circulant* $W = W(q^2 + q + 1, q^2)$. *Let* $p \equiv 1 (mod\ 4)$ *then there are two circulant symmetric matrices* R, S *of order* $(p+1)/2$ *satisfying*

$$RR^T + SS^T = pI.$$

Lemma 4 **[6, Proof of Lemma 4.34]** *Let* q *be a prime. Then there is a circulant matrix* Q *which satisfies* $QQ^T = qI - J$, $QJ = JQ = 0$, $Q^T = (-1)^{(q-1)/2} Q$.

Corollary 2 *There exists a circulant* $W = W(13,9)$. *There exist two circulant symmetric matrices* R *and* S *or order 13 satisfying* $RR^T + SS^T = 25I$. *There exists a circulant symmetric matrix* Q *of order 13 satisfying* $QQ^T = 13I - J$.

Lemma 5 [6, Lemmas 4.21 and 4.22] *Let* A *and* B *be circulant matrices of order* n *and* $R = (r_{ij})$ *where* $r_{ij} = 1$ *if* $i + j - 1 = n$ *and 0 otherwise, then* $A(BR)^T = (BR)A^T$.

3 Notation

I is the identity matrix with the order taken from the context;

J is the matrix of ones with the order taken from the context;

X is the backcirculant matrix with first row $\{a\ b\ 0_{10}\ \bar{b}\}$ where 0_{10} is a sequence of 10 zeros and a and b are commuting variables;

Y is the circulant matrix with first row $\{0\ b\ 0_{10}\ b\}$ where 0_{10} is a sequence of 10 zeros and b is a commuting variable;

W is the backcirculant matrix with first row $\{0\ 1\ 0\ 1\ 1\ 0\ 0\ -\ -\ 1\ 1\ -\ 1\}$ where $-$ is used for -1, and W is a $W(13,9)$;

R and S are circulant symmetric matrices satisfying $RR^T + SS^T = 25I$;

Q is the circulant symmetric matrix of order 13 satisfying $QQ^T = 13I - J$;

A, B, C, D are circulant symmetric matrices satisfying $AA^T + BB^T + CC^T + DD^T = 52I$ (these are Williamson matrices see [12, pp511, 541].

$I + K$, L, M, N are circulant matrices where K is skew-symmetric, $(cI + dK)'$ is the backcirculant matrix with the same first row as $cI + dK$, and L, M and N are symmetric satisfying $KK^T + LL^T + MM^T + NN^T = 51I$ (these are good matrices see [12, pp492].

4 Sequences with Zero Autocorrelation

Tables 1 to 4 give sequences of lengths 13 and 15 with zero non-periodic and periodic auto-correlation function.

Length=13	Sequences with zero non-periodic autocorrelation function
1,34	{+ 0 0 + − − *a* + + − 0 0 −}, {0 0 0 + + + + 0 + − + − +},
	{0 − 0 + 0 + + 0 + 0 0 − +}, {− 0 0 − + + − 0 + + + − −}
1,37	{0 + − + + 0 *a* 0 − − + − 0}, {+ + − + − + 0 + + + + + −},
	{− + 0 − + + 0 − − + + 0 0}, {− − − 0 + 0 + 0 0 0 + + −}
1,1,40	{+ 0 + + + − *a* + − − − 0 −}, {+ 0 + − − + *b* − + + − 0 −},
	{+ 0 + + + − 0 − + + + 0 +}, {+ 0 + − − + 0 + − − + 0 +}
1,45	{+ + + − − + *a* − + + − − −}, {+ + − + − + 0 + − + + + +},
	{− + − + + − + + + + − 0 0}, {+ + − + + 0 − − − − + 0 0}
48	{+ + + − + + + + − + − + 0}, {+ + + − + + − − + − + − 0}
	{+ + + − − − + + − + + − 0}, {+ + + − − − − − + − − + 0}

Table 1: Sequences of length 13 with zero non-periodic autocorrelation function

Length=13	Sequences with zero periodic autocorrelation function
1,42	{− + + − − 0 *a* 0 + + − − +}, {− + − 0 − + 0 + + + + + +},
	{+ + + + − + − 0 + 0 0 + −}, {+ + − − − + 0 0 + + + − −}
1,46	{− − − + + − *a* + − − + + +}, {− + + + − + 0 + + + − + +},
	{+ + + + + + 0 − − 0 − + −}, {+ + − − 0 − + 0 + + − + −}
1,48	{+ + + − + + *a* − − + − − −}, {− + + + + + 0 − + + − + −},
	{− + 0 + − + − − + + + + +}, {+ + + + − + − − + + 0 + −}
1,49	{+ + − − + − *a* + − + + − −}, {+ + + + + + 0 + − + − + −}
	{+ + − − + + + + + + − − −}, {+ + + + − − 0 − + + − + −}

Table 2: Sequences of length 13 with zero periodic autocorrelation function

Length=15	Sequences with zero non-periodic autocorrelation function
49	{− + 0 + + 0 0 + + + 0 + − 0 +}, {+ − − − + − + + + + − + + + −}
	{− 0 + + 0 + + − 0 0 − + 0 + −}, {+ − − − − + + 0 + − + + − − +}
1,53	{0 + + + − + + *a* − − + − − − 0}, {+ + + + 0 − + + + − + − + + −},
	{+ − + + − + + − + − 0 − + + +}, {+ − − − + + + + + − − + − 0 0}
1,56	{+ − + − − − − *a* + + + + − + −}, {− + − + + + + 0 + + + + − + −},
	{+ + − − + + + + + − − + + − 0}, {+ − − + + − + − + + − − + + 0}

Table 3: Sequences of length 15 with zero non-periodic autocorrelation function

Length=15	Sequences with zero periodic autocorrelation function
1,42	{0 + 0 − + − − *a* + + − + 0 − 0}, {0 + − + − + − − 0 + + + + + +},
	{+ + − 0 0 + + 0 − + + 0 0 + −}, {+ + 0 0 0 − 0 − − + + + 0 0 −}
1,54	{+ + + − − + − *a* + − + + − − −}, {+ + − 0 + − − + + + − + + + 0},
	{0 + + + − + − + + + − 0 + − +}, {− + + + + + + 0 − + − + − − −}
1,57	{− − + − + − − *a* + + − + − + +}, {+ + − + − + − − + + − + + + +},
	{− + + + + + − − + + + 0 − + −}, {+ + + 0 − + − − − + + + + + −}

Table 4: Sequences of length 15 with zero periodic autocorrelation function

Length=17	Sequences with zero non-periodic autocorrelation function
63	$\{+-+-+-++\ 0+++++++--\}$,
	$\{++-+-++-++-++--++\}$,
	$\{0-++-0----+++-+++\}$,
	$\{+-++++---+++0-+--0\}$,

Table 5: Sequences of length 17 with zero non-periodic autocorrelation function

Length=17	Sequences with zero periodic autocorrelation function
1,61	$\{---0++-+a-+--0+++\}$,
	$\{-+-++++++++++-+--\ 0\}$,
	$\{++-+-+++-++-++--\ 0\}$,
	$\{++--\ 0+-+++-0-++-+\}$,
1,65	$\{a+--+-----++++-++-\}$,
	$\{++--+++-++++-+-++\}$,
	$\{0+++-+-+-+++-++--\}$,
	$\{0-++------+-++++-+\}$

Table 6: Sequences of length 17 with zero periodic autocorrelation function

Length=18	Sequences with zero non-periodic autocorrelation function
1,66	$\{0+++-+-++a--+-+---\}$,
	$\{0+++-+-++0++-+-+++\}$,
	$\{+++-\ 0+---+++--+--+\}$,
	$\{+++-\ 0+-------++-++-\}$

Table 7: Sequences of length 18 with zero non-periodic autocorrelation function

5 Results in Orders Divisible by 13

We recall that orthogonal designs $OD(52;1,k)$ can only exist if k is the sum of three squares. We see $52-2 = 5^2 + 5^2 = 7^2 + 1^2$ so the other condition is satisfied. Hence we have that $OD(52;1,k)$ cannot exist for $k = 4^a(8b+7)$, ie $k \in \{7, 15, 23, 28, 31, 39, 47\}$.

Theorem 6 *Orthogonal designs $OD(52;1,k)$ exist for $k \in \{x : x = a^2 + b^2 + c^2\}$. In other words the necessary conditions are sufficient for the existence of an $OD(52;1,k)$. All are constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. From [6, Theorem 4.149] we get the result for $k \neq 34, 37, 42, 45, 46, 48$ or 49. Tables 1 and 2 give 4 sequences which can be used in Corollary 1 to give all these values.

Corollary 3 *$W(52,k)$ exist for all $k = 1, 2, \ldots, 52$.*

Proof. From the theorem we only have to consider $k \in \{7, 15, 23, 28, 31, 39, 47\}$ as all other values of k have an $OD(52;1,k)$: setting the first variable zero gives the required weighing matrix. For these other values we consider $OD(52;1,k-1)$ and equate the variables to give the result. □

Corollary 4 *Orthogonal designs $OD(104;1,k)$ exist for $k = 1, 2, \ldots, 103$ with the possible exception of 94 and 95 which are undecided.*

Proof. We use Lemma 1 to construct $OD(104;1,1,k,k)$ for k given in the previous Theorem. This assures us of the existence of all $OD(1,j)$ with the possible exception of $j = 56, 57, 62, 63, 78, 79, 94$ and 95. We replace the variables of the $OD(8;1,1,1,1,1,1,1,1)$ as given in Table 8 to get the orthogonal designs indicated there:

Variables Replaced By								Design Constructed
cI	dI	X	Y	eA	eB	eC	eD	OD(104;1,1,1,4,52)
aI	bI	cI	dW	eA	eB	eC	eD	OD(104;1,1,1,9,52)
aI	bI	cS	cR	dA	dB	dC	dD	OD(104;1,1,25,52)
X	Y	cS	cR	dS	dR	eS	eR	OD(104;1,4,25,25,25)

Table 8: Construction of Orthogonal Designs in Order 104.

So by equating variables and setting variables to zero we have constructed $OD(104;1,i)$, for $i = 56, 57, 62, 63, 78$ and 79 giving the result. □

Corollary 5 *Orthogonal designs $OD(208;1,k)$ exist for $k = 1, 2, \ldots, 207$ with the possible exception of 189 and 191 which are undecided. All $W(208,k)$ exist, $k = 1, 2, \ldots, 208$.*

Proof. We use Lemma 1 to construct $OD(208;1,1,k,k)$ for k given in the previous Corollary. This assures us of the existence of all $OD(1,j)$ with the possible exception of $j = 188, 189, 190$ and 191. We replace the variables of the $OD(16;1,1,1,1,1,1,5,5)$ by aI, bW, cQ, $dI + cQ$, $dI - cQ$, cJ, $eI + cQ$, $eI - cQ$ to obtain an $OD(208;1,2,9,10,169)$ and hence equating and killing variables the $OD(208;1,i)$, $i = 188$ and 190 giving the result. □

Corollary 6 *Orthogonal designs $OD(416;1,k)$ exist for $k = 1, 2, \ldots, 415$. All $W(416,k)$ exist, $k = 1, 2, \ldots, 416$.*

Proof. We use Lemma 1 to construct $OD(416;1,1,k,k)$ for k given in the previous Corollary. This assures us of the existence of all $OD(1,j)$ with the possible exception of 378, 379, 382 and 383. We replace the variables of the following designs in order 32 (i) OD(32;1,1,3,3,3,3,9,9) by aI, bI, $(cI + dK)'$, dL, dM, dN, eR and eS to obtain the $OD(416;1,1,3,153,225)$ giving the result for 378, 379 and 382, and (ii) OD(32;1,1,1,1,2,2,3,3,9,9) by aI, bI, $dI + cQ$, $dI - cQ$, $c(J - I)$, cQ, eR, eS, $fI + cQ$ and $fI - cQ$ to obtain the $OD(416;1,1,2,18,75,288)$ design which gives by equating variables the $OD(416;1,1,2,381)$ giving the result for 383. □

Hence using Lemma 3 we have

Theorem 7 *Orthogonal designs* $OD(2^t.13; 1, k)$ *exist for* $k = 1, 2, \ldots, 2^t.13 - 1$ *for all* $t \geq 5$. *All* $W(2^t.13, k)$ *exist,* $k = 1, 2, \ldots, 2^t.13$ *for all* $t \geq 5$.

6 Results in Orders Divisible by 15

We recall that orthogonal designs $OD(60; 1, k)$ can only exist if k is the sum of three squares. We see $60-2 = 7^2 + 3^2$ so the other condition is satisfied. Hence we have that $OD(60; 1, k)$ cannot exist for $k = 4^a(8b+7)$, ie $k \in \{7, 15, 23, 28, 31, 39, 47, 55\}$.

Theorem 8 *Orthogonal designs* $OD(60; 1, k)$ *exist for* $k \in \{x : x = a^2 + b^2 + c^2\}$ *except possibly for* k = *48 or 49 which are undecided. In other words the necessary conditions are sufficient for the existence of an* $OD(60; 1, k)$ *except possibly for* k = *48 or 49 which are undecided. All, except the* $OD(60; 1, 46)$, *are constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. From [6, Theorem 4.149] we have the result for $k \neq 34, 37, 42, 45, 46, 48, 49, 53, 54, 56$ or 57. Tables 1,3 and 4 give 4 sequences which can be used in Corollary 1 to give all these values except 46, 48 and 49. We replace the variables of the $OD(12; 1, 1, 5, 5)$ by aI, bI, $c(J-2I)$, dQ to give the $OD(60; 1, 1, 45)$ and hence the $OD(60; 1, 46)$.

Corollary 7 $W(60, k)$ *exist for all* $k = 1, 2, \ldots, 60$.

Proof. From the theorem we only have to consider $k \in \{7, 15, 23, 28, 31, 39, 47, 48, 49, 55\}$ as all other values of k have an $OD(60; 1, k)$: setting the first variable zero gives the required weighing matrix. The sequences that can be used to give weights 48 and 49 are given in Tables 1 and 3 (note that for sequences with zero non-periodic autocorrelation function the appropriate number of zeros can be added to the end of each sequence to give the required length). For the other values we consider $OD(60; 1, k-1)$ and equate the variables to give the result. □

Corollary 8 *Orthogonal designs* $OD(120; 1, k)$ *exist for* $k = 1, 2, \ldots, 119$. *All* $W(120, k)$, $k = 1, 2, \ldots, 120$ *exist.*

Proof. We use Lemma 1 to construct $OD(120; 1, 1, k, k)$ for k given in the previous Theorem. This assures us of the existence of all $OD(1, j)$ with the possible exception of $j = 47, 62, 63, 78, 79, 94, 95, 96, 97, 98, 99, 110$, and 111.

I_n is the identity matrix with the order n taken from the context;

J_n is the matrix of ones with the order n taken from the context;

Write $K = J - 2I$ and $L = J - I$;

X is the backcirculant matrix with first row {a b 0 0 $\bar{b}$ } where a and b are commuting variables;

Y is the circulant matrix with first row {0 b 0 0 b} where b is a commuting variable;

A is the backcirculant matrix with first row {a b $\bar{b}$ } where a and b are commuting variables;

B is the backcirculant matrix with first row {a b b $\bar{b}$ $\bar{b}$ } where a and b are commuting variables;

Q is the circulant symmetric matrix of order 5 with first row {0 1 - - 1} satisfying $QQ^T = 5I - J$;

$I + E$, F, G, H are circulant matrices where E is skew-symmetric, $(cI + dE)'$ is the backcirculant matrix with the same first row as $cI + dE$, and F, G and H are symmetric satisfying $EE^T + FF^T + GG^T + HH^T = 19I$ (these are good matrices see [12, pp492].

We replace the variables of the indicated OD in orders 24 and 40 as given in Table 9 to get the orthogonal designs indicated there:

Variables In	Variables Replaced By						Design Constructed
OD(24;4,4,1,1,5,5)	eJ-2eI	eQ	aI	bI	cI	dI	OD(120;1,1,5,5,36)
OD(24;4,4,1,5,5,1)	eJ-2eI	eQ	cI	X	Y	dI	OD(120;1,1,5,20,36)
OD(24;6,1,1,1,9,6)	eJ-2eI	aI	bI	cI	dI	eQ	OD(120;1,1,1,9,54)
OD(24;4,4,4,4,1,2)	$(fI$+e$E)'$	eF	eG	eH	aI	bI	OD(120;1,2,4,76)
OD(24;4,4,4,4,1,3)	$(fI$+e$E)'$	eF	eG	eH	aI	bI	OD(120;1,3,4,76)
OD(24;9,1,12,1,1)	cJ-2cI	cJ-cI	cQ	aI	bI		OD(120;1,1,97)
OD(24;5,1,1,2,15)	cJ-2cI	X	Y	cJ	cQ		OD(120;1,4,95)
OD(24;9,9,1,1,1,3)	dI+cQ	dI-cQ	cL	aI	bI	cJ	OD(120;1,1,18,91)
OD(24;1,2,12,8,1)	bI+bQ	bI-bQ	cK	bQ	B		OD(120;1,111)
OD(40;18,19,1)	bJ-2bI	bJ-bI	A				OD(120;1,94)
OD(40;19,19,1,1)	bJ-2bI	bJ-bI	aI	cI			OD(120;1,1,95)

Table 9: Construction of Orthogonal Designs in Order 120.

Setting variables equal to each other or to zero gives all the remaining cases. □

Hence using Lemma 3 we have

Theorem 9 *Orthogonal designs* $OD(2^t.15; 1, k)$ *exist for* $k = 1, 2, \ldots, 2^t.15 - 1$ *for all* $t \geq 3$. *All* $W(2^t.15, k)$ *exist,* $k = 1, 2, \ldots, 2^t.15$, $t \geq 3$.

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