Characterizations of Various Matching Extensions in Graphs

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Abstract

Let n be a positive integer with $n \leq (V(G)-2)/2$. A graph G is n-**extendable** if it contains a set of n independent edges and every set of n independent edges can be extended to a perfect matching of G. In this paper, we give a characterization of n-extendable graphs. The characterizations of other matching extension are also discussed.

1. Introduction

All graphs in this paper are finite and have no loops or multiple edges.

A **perfect matching**, or 1-**factor**, of a graph G is a set of independent edges which together cover all the vertices of G. Let n be a positive integer with $n \leq \frac{V(G)-2}{2}$. A graph G is n-**extendable** if it contains a set of n independent edges and every set of n independent edges can be extended to a perfect matching of G. We call G 0-**extendable** if it has a perfect matching. A graph G is said to be **bicritical** if for every pair of distinct vertices u and v $G-\{u, v\}$ has a perfect matching (clearly bicritical graphs are 1-extendable). A 3-connected bicritical graph is called a **brick**. A graph G is said to be **factor-critical** if G-v has a perfect matching for every $v \in V(G)$.

In 1980, Plummer [7] studied the properties of n-extendable graphs and showed that every 2-extendable graph is either bipartite or a brick. Motivated by this result he [8, 9] further looked at the relationship between n-extendability and other graphic parameters (e.g., degree, connectivity, genus, toughness). Recently, Schrag and Cammack [11] and Yu [12] classified the 2-extendable generalized Petersen graphs, and Chan, Chen and Yu [3] classified the 2-extendable Cayley graphs on abelian groups. For more results and the motivations of n-extendable graphs, the interested reader is refereed to a recent survey paper by Plummer [10].

Little, Grant and Holton [4] gave good characterizations of 1-extendable graphs and 1-extendable bipartite graphs. Brualdi and Perfect [2] in 1971 obtained a criterion of n-extendable bipartite graphs, but their result is described in terms of matrices and systems of distinct representatives. In this paper, we shall characterize the n-extendable graphs ($n \geq 1$). Since n-extendable graphs must have a 1-factor, we deal only with graphs of even order. For graphs of odd order, we generalize the idea of n-extendability and introduce $n\frac{1}{2}$-extendability. A graph G is **$n\frac{1}{2}$-extendable** if (1) for any vertex v of V(G) there exists a set of n independent edges in G which miss v and (2) for every vertex v and every set of n independent edges $e_1 = x_1y_1$, $e_2 = x_2y_2$, ..., $e_n = x_ny_n$ missing v, there exists a near perfect matching of G which contains e_1, e_2, ..., e_n and misses v. Analogous to n-extendability, we study the properties of $n\frac{1}{2}$-extendable graph and obtain a characterization for it. The generalizations of factor-critical and bicritical graphs are also discussed.

For any set $S \subseteq V(G)$, we denote by G-S the subgraph of G obtained by deleting the vertices of S together with their incident edges, and by G[S] the subgraph of G induced by S.

The followings are some preliminary results which we need in this paper.

Theorem 1.1 (Tutte's Theorem) A graph G has a perfect matching if and only if $o(G-S) \leq |S|$ for all $S \subseteq V(G)$.

Theorem 1.2 (Little, Grant and Holton [4]) Let G be a graph of even order. Then G is 1-extendable if and only if for all $S \subseteq V(G)$,

(1) $o(G-S) \leq |S|$ and

(2) $o(G-S) = |S|$ implies that S is an independent set.

Theorem 1.3 (Plummer [7]) If G is a graph with p vertices, then the following claims hold.

(1) If G is n-extendable, then G is also (n-1)-extendable.

(2) If G is a connected n-extendable graph, then G is (n+1)-connected.

(3) If $p \geq 4$ and $d(G) \geq \frac{p}{2} + n$, then G is n-extendable.

Theorem 1.4 (See [6]) A graph G is factor-critical if and only if G has an odd number of vertices and $o(G-S) \leq |S|$ for all $\emptyset \neq S \subseteq V(G)$.

2. Characterizations and Properties

The family of n-extendable graphs is quite large. For example, the cube, the tetrahedron, the dodecahedron and the complete bipartite graph $K_{r,r}$ are 2-extendable. In fact, if the minimum degree $\delta(G)$ is larger than $n+|V(G)|/2$ and $|V(G)| \geq 4$, then G is n-extendable (see Theorem 1.3 (3)).

Several results in this section will be based on the following observation.

Observation 2.1 A graph G is n-extendable if and only if for any matching M of size i $(1 \leq i \leq n)$ the graph G-V(M) is (n-i)-extendable.

Proof: Suppose that G is n-extendable. For any matching M of size i $(1 \leq i \leq n)$, let H = G-V(M). Observe that by Theorem 1.3 (1) H has a perfect matching. Let M' be a matching of H with n-i edges. Then $M \cup M'$ is an n-matching of G and thus there exists a perfect matching P of G containing $M \cup M'$. Clearly, P-M is a perfect matching of H which contains M' and so H is (n-i)-extendable.

Conversely, for any matching Q of size n in G, let M be a subset of Q with i edges. By assumption G-V(M) is (n-i)-extendable. Thus there exists a perfect matching P of G-V(M) containing Q-M and therefore $P \cup M$ is a perfect matching of G containing Q. □

We begin by giving a characterization of n-extendable graphs which is a generalization of Theorem 1.2.

Theorem 2.2 A graph G is n-extendable $(n \geq 1)$ if and only if for any $S \subseteq V(G)$

(1) $o(G-S) \leq |S|$ and

(2) $o(G-S) = |S|-2k$ $(0 \leq k \leq n-1)$ implies that $F(S) \leq k$, where F(S) is the size of a maximum matching in G[S].

Proof: Suppose G is n-extendable. Since G has a perfect matching, (1) follows from Tutte's theorem. Suppose $o(G-S) = |S|-2k$ $(0 \leq k \leq n-1)$ for some vertex-set $S \subseteq$

$V(G)$. We consider first the case that $k = n-1$. In this case, assume $F(S) > n-1$. Let $e_i = x_iy_i$ $(1 \le i \le n-1)$ be $n-1$ independent edges in $G[S]$. By Observation 3.1, $G-\{x_1, y_1, ..., x_{n-1}, y_{n-1}\}$ is 1-extendable. Let $G' = G-\{x_1, y_1, ..., x_{n-1}, y_{n-1}\}$ and $S' = S-\{x_1, y_1, ..., x_{n-1}, y_{n-1}\}$. Then $o(G'-S') = o(G-S) = |S|-2(n-1) = |S'|$. By Theorem 1.2, S' is an independent set. Thus $F(S) \le F(S')+(n-1) = n-1 = k$, a contradiction. Since k-extendability implies (k-1)-extendability, (2) holds for $0 \le k \le n-2$.

Now suppose (1) and (2) hold. The proof that G is n-extendable will use induction on n.

If $n = 1$, the claim holds from Theorem 1.2 as $F(S) = 0$ means that S is independent.

Suppose that the claim holds for $n < r$. Consider $n = r$. By the induction hypothesis, (1) and (2) imply that G is (r-1)-extendable. If G is r-extendable, we are done. Otherwise, there exist r-1 independent edges $e_i = x_iy_i$ $(1 \le i \le r-1)$ so that $G' = G-\{x_1, y_1, ..., x_{r-1}, y_{r-1}\}$ is not 1-extendable. Since G' has a perfect matching, condition (1) of Theorem 1.2 holds. Thus, if G' is not 1-extendable, then there exists a set $S' \subseteq V(G')$ so that $o(G'-S') = |S'|$ and $F(S') \ge 1$. Let $S = S' \cup \{x_1, y_1, ..., x_{r-1}, y_{r-1}\}$. Then $o(G-S) = o(G'-S') = |S'| = |S|-2(r-1)$ and $F(S) \ge F(S')+(r-1) \ge r$, which contradicts condition (2). □

Next we study relationships between n-extendability and $n\frac{1}{2}$-extendability. It turns out that they are very similar. If a new vertex is joined to all vertices of an $n\frac{1}{2}$-extendable graph G, then the resulting graph is (n+1)-extendable. Thus (n+1)-extendable graphs can be obtained by this method and in this sense, $n\frac{1}{2}$-extendability is weaker than (n+1)-extendability. On the other hand, if G is $n\frac{1}{2}$-extendable, then for any vertex $v \in V(G)$, G-v is n-extendable. Hence $n\frac{1}{2}$-extendability is "stronger" than n-extendability. However, there exist (n+1)-extendable graphs with the property that on deletion of some vertex the resulting graph is not $n\frac{1}{2}$-extendable; for example, the cube is 2-extendable but on deleting any vertex v, G-v is not $1\frac{1}{2}$-extendable. So it is natural to think of $n\frac{1}{2}$-extendability as lying between n and (n+1)-extendability. Not surprising then, we can characterize all $n\frac{1}{2}$-extendable graphs in terms of n-extendable and (n+1)-extendable graphs.

Theorem 2.3 A graph G of odd order is $n\frac{1}{2}$-extendable if and only if $G+K_1$ is (n+1)-extendable.

Proof: Assume that G is $n\frac{1}{2}$-extendable. Let $H = G+K_1$, where $V(K_1) = \{z\}$ and choose n+1 independent edges, $e_i = x_iy_i$ ($i = 1, 2, ..., n+1$) of E(H).

Case 1. All n+1 independent edges lie in E(G). Since G is $n\frac{1}{2}$-extendable, there exists a near perfect matching M containing $e_1, e_2, ..., e_n$ and missing x_{n+1} in G. Let w be the vertex adjacent to y_{n+1} in M. Then $M-\{wy_{n+1}\}\cup\{wz, x_{n+1}y_{n+1}\}$ will be a perfect matching of H containing $e_1, e_2, ..., e_{n+1}$.

Case 2. Suppose that one of $e_1, e_2, ..., e_{n+1}$ is not in E(G), say e_{n+1}. Let e_{n+1} = zw, where $w \in V(G)-\{x_1, y_1, ..., x_n, y_n\}$. Then there exists a near perfect matching M of G containing $e_1, e_2, ..., e_n$ and missing the vertex w. Thus $M\cup\{zw\}$ is a perfect matching of H as required.

Conversely, for any n independent edges $e_1, e_2, ..., e_n$ of E(G) and vertex v of V(G) not lying on these edges, there exists a perfect matching M of H containing e_1, $e_2, ..., e_n$, vz. Then $M' = M-\{z\}$ is a near perfect matching of G which contains e_1, e_2, $..., e_n$ and misses v. □

Remark: Even though when G is $n\frac{1}{2}$-extendable, $G+K_1$ is (n+1)-extendable, it is not the case that if G is n-extendable, then $G+K_1$ is $n\frac{1}{2}$-extendable. For example, the cycle C_{2m} is 1-extendable, but $C_{2m}+K_1$ is not $1\frac{1}{2}$-extendable

From the definition of $n\frac{1}{2}$-extendability, we have the following observation.

Observation 2.4 A graph G is $n\frac{1}{2}$-extendable if and only if G-v is n-extendable for any vertex $v\in V(G)$.

We now give a characterization of $n\frac{1}{2}$-extendable graphs.

Theorem 2.5 A graph G is $1\frac{1}{2}$-extendable if and only if for any $S \subseteq V(G)$, $S \neq \emptyset$,

(1) $o(G-S) \leq |S|-1$ and

(2) if both $o(G-S) = |S|-1$ and $|S| \geq 3$, then S is independent.

Proof: If G is $1\frac{1}{2}$-extendable, then G is factor-critical, and by Theorem 1.4 condition (1) holds.

Suppose there exists a vertex-set S of V(G) with $|S| \geq 3$ such that $o(G-S) = |S|-1$ but S is not independent. Let $e = xy\in E(G[S])$ and $z\in S-\{x,y\}$. Let $G' = G-\{z\}$ and $S' = S-\{z\}$. Then, as by Observation 2.4 G' is 1-extendable, it follows that o(G'-

S') = o(G-S) = |S|-1 = |S'|. From Theorem 1.2, S' must be an independent set. But this contradicts the fact that $e \in E(G[S'])$.

Conversely, condition (1) guarantees that G has an odd number of vertices (choose S = {v}, $v \in V(G)$) and then Theorem 1.4 implies that G is factor-critical. But we need the stronger result that G-{v} is 1-extendable for any $v \in V(G)$. Suppose that for $v \in V(G)$ and $e \in E(G-v)$ there is no perfect matching in G-v containing e. Since G-v has a perfect matching, then by Theorem 1.2 and Theorem 1.1 we know that there exists a vertex-set $S \subseteq V(G-v)$ so that o(G-v-S) = |S| and S is not independent. Thus $|S| \geq 2$. Let $S'' = S \cup \{v\}$. Then o(G-S") = o(G-v-S) = |S| = |S"|-1 and $|S''| \geq 3$, but S" is not independent. This contradicts condition (2). □

Theorem 2.6 A graph G is $n\frac{1}{2}$-extendable if and only if for any $S \subseteq V(G)$, $S \neq \varnothing$,

(1) $o(G-S) \leq |S|-1$ and

(2) if $o(G-S) = |S|-2k-1$ ($0 \leq k \leq n-1$) and $|S| \geq 2k+3$ for some vertex-set $S \subseteq V(G)$, then $F(S) \leq k$, where F(S) is the size of maximum matching in G[S].

Proof: The proof will be by induction on n. When n = 1, it is Theorem 2.5.

Suppose the theorem holds when n < r, and consider the case n = r.

Assuming that G is $r\frac{1}{2}$-extendable, it follows that G is factor-critical. Thus (1) follows from Theorem 1.4. If $o(G-S) = |S|-2k-1$ ($0 \leq k \leq r-2$) and $|S| \geq 2k+3$, then by the induction hypothesis, $F(S) \leq k$. Suppose then that there exists a set S such that $o(G-S) = |S|-2(r-1)-1$ and $|S| \geq 2r+1$ ($k = r-1$), but $F(S) \geq r$. Let $e_i = x_i y_i$ ($1 \leq i \leq r$) be r independent edges in G[S], $v \in S' = S-\{x_1, y_1, ..., x_r, y_r\}$ and $G' = G-\{x_1, y_1, ..., x_r, y_r, v\}$. Then $o(G'-S') = o(G-S) = |S|-2r+1 = |S'|+2 > |S'|$ and by Tutte's theorem, G' has no perfect matching. This contradicts the fact that G is $r\frac{1}{2}$-extendable.

Conversely, suppose that conditions (1) and (2) hold but G is not $r\frac{1}{2}$-extendable. Then there exists a vertex $v \in V(G)$ such that G-v is not r-extendable. Applying Observation 2.1, there exist independent edges $e_i = x_i y_i$ ($1 \leq i \leq r-1$) so that $G' = G-v-\{x_1, y_1, ..., x_{r-1}, y_{r-1}\}$ is not 1-extendable. However, from the induction hypothesis G is $(r-1)\frac{1}{2}$-extendable and thus G' has a perfect matching. Then from Tutte's Theorem for all $S \subseteq V(G')$, $o(G'-S) \leq |S|$. But now as G' is not 1-extendable, from Theorem 1.2, there exists a set $S' \subseteq V(G')$ such that $o(G'-S') = |S'|$ and S' is not independent. Let $S = S' \cup \{v, x_1, y_1, ..., x_{r-1}, y_{r-1}\}$. Then $o(G-S) = o(G'-S') = |S'| = |S|-2(r-1)-1 = |S|-2r+1$ and so $|S| = |S'|+2(r-1)+1 \geq 2+2(r-1)+1 = 2r+1$. But $F(S) \geq F(S')+(r-1) \geq r$, which contradicts condition (2) when k = r-1. □

Corollary 2.7 If G is an $n\frac{1}{2}$ -extendable graph, then G is also $(n-1)\frac{1}{2}$ -extendable.

We now turn to study some of the properties of $n\frac{1}{2}$ -extendable graphs. They are analogous to those of n-extendable graphs.

Theorem 2.8 If G is a graph of order 2r+1, $r \geq n+1 \geq 2$ and $\delta(G) \geq r+n+1$, then G is $n\frac{1}{2}$ -extendable. Moreover, the lower bound on $\delta(G)$ is sharp.

Proof: By Observation 2.4, we need only to show that for any $v \in V(G)$ G-v is n-extendable. For any $v \in V(G)$, $\delta(G-v) \geq \delta(G)-1 \geq r+n$. From Theorem 1.3 (3), G-v is n-extendable and we are done.

To see that the bound is sharp, consider the graph $G = K_{r+n} + \overline{K}_{r-n+1}$. Since $r \geq n+1$, we take a vertex v and n independent edges x_1y_1, x_2y_2, ..., x_ny_n from K_{r+n}. There remain r-n-1 vertices in K_{r+n} which cannot be matched to the r-n+1 vertices in $\overline{K}_{r-n+1}$. Thus $\delta(G) = r+n$ and G is not $n\frac{1}{2}$ -extendable. □

Theorem 2.9 If G is connected and $n\frac{1}{2}$ -extendable ($n \geq 1$), then G is (n+2)-connected and, moreover, there exists an $n\frac{1}{2}$ -extendable graph G of connectivity n+2.

Proof: If G is $n\frac{1}{2}$ -extendable, then, by Theorem 2.3, $G+K_1$ is (n+1)-extendable. Since $G+K_1$ is connected, by Theorem 1.3 (2), $G+K_1$ is (n+2)-connected. Let $K_1 = \{u\}$. Since $n \geq 1$, $G-v = (G+K_1)-\{u, v\}$ is connected for any $v \in V(G)$. By Observation 2.4, G-v is n-extendable for any $v \in V(G)$. Thus G-v is (n+1)-connected by applying Theorem 1.3 (2).

Suppose that G is not (n+2)-connected. Then there exists a cut-set $S \subseteq V(G)$, $|S| = n+1$. For any $v \in S$, $S-\{v\}$ is a cut-set of G-v. Since $|S-\{v\}| = n$, this contradicts the fact that G-v is (n+1)-connected.

To see that an $n\frac{1}{2}$ -extendable graph might not be (n+3)-connected, we consider the graph $G = \overline{K}_{n+2}+(K_p \cup K_q)$ where n+2+p+q is odd and $p \geq q \geq 2n+2$. Clearly G is not (n+3)-connected as $V(\overline{K}_{n+2})$ is a cut-set of size n+2. We next show that G is $n\frac{1}{2}$ -extendable. For any given n independent edges $e_i = x_iy_i$, $1 \leq i \leq n$, and a vertex $v \notin \{x_1, y_1, x_2, y_2, ..., x_n, y_n\}$, let $S = \{v, x_1, y_1, x_2, y_2, ..., x_n, y_n\}$, $V_1 = V(K_p)-S$, $V_2 = V(\overline{K}_{n+2})-S$ and $V_3 = V(K_q)-S$ (see Figure 2.1). We now need

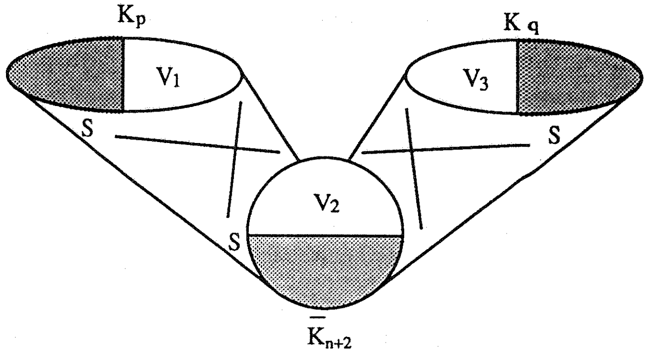

Figure 2.1

only to show that G-S has a perfect matching. Clearly, the existence of a perfect matching in the graph G-S is equivalent to a partition of V_2 into two subsets V_2', V_2'' such that $|V_2'| \leq |V_1|$, $|V_2''| \leq |V_3|$, $|V_2'| \equiv |V_1| \pmod 2$, and $|V_2''| \equiv |V_3| \pmod 2$. As $|V(G)|$ is odd and p, q $\geq$ 2n+2, we have that $|V_1|+|V_2|+|V_3| = |V(G)|-|S| = p+q+1-n$ is even and $|V_1|+|V_3| \geq |V_2|+2$. Therefore the required partition (V_2', V_2'') can always be achieved. This completes the proof. □

Remark: Theorem 2.9 does not hold for n = 0; that is, for factor-critical graphs. The graph below provides an example of a $\frac{1}{2}$ -extendable graph which is not 2-connected.

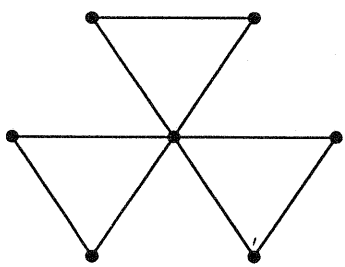

Figure 2.2 The factor-critical graph is not 2-connected.

Corollary 2.10 If G is an $n\frac{1}{2}$ -extendable graph of order p, p $\geq$ 2n+5, and if u is a vertex of degree n+2 in G, then $N_G(u)$ is an independent set.

Proof: Suppose u is a vertex of degree n+2 in an $n\frac{1}{2}$-extendable graph G and let $N_G(u) = \{v_1, v_2, ..., v_{n+2}\}$. Since $p > 2n+4$, we can choose n+1 vertices $w_1, w_2, ..., w_{n+1}$ in $V(G)-N_G(u)-\{u\}$. As G is (n+2)-connected, by Menger's theorem we have n+2 vertex-disjoint paths joining $N_G(u)$ and $\{w_1, w_2, ..., w_{n+1}, u\}$. Hence there are n+2 independent edges $e_1 = v_1u$, $e_2 = v_2w_1'$, ..., $e_{n+2} = v_{n+2}w_{n+1}'$, where w_i' is the last vertex on the path from w_i to v_{i+1}.

Suppose now that $N_G(u)$ is not independent, say $v_1v_2 \in E(G)$. Then $v_1v_2, e_4, e_5, ..., e_{n+2}$ are n independent edges. Since u is an isolated vertex of $G-N_G(u)$, there exists no near perfect matching containing $v_1v_2, e_4, e_5, ..., e_{n+2}$ and missing v_3. This contradicts the fact that G is $n\frac{1}{2}$-extendable. □

A graph G is called **n-critical** if the deletion of any n vertices of V(G) results in a graph with a perfect matching. This concept is a generalization of the notions of factor-critical and bicritical which correspond to the cases when n = 1 and n = 2, respectively. Here we present a characterization of n-critical graphs.

Theorem 2.11 A graph G is n-critical if and only if $|V(G)| \equiv n \pmod 2$ and for any vertex-set $S \subseteq V(G)$ with $|S| \geq n$, $o(G-S) \leq |S|-n$.

Proof: Suppose that G is n-critical. Then it is immediate that $|V(G)| \equiv n \pmod 2$. Suppose there is a vertex-set $S \subseteq V(G)$ with $|S| \geq n$ and $o(G-S) > |S|-n$. Delete n vertices $v_1, v_2, ..., v_n$ from S and denote the remaining set by S'. Then $o(G-\{v_1, v_2, ..., v_n\}-S') = o(G-S) > |S|-n = |S'|$ and by Tutte's theorem, $G-\{v_1, v_2, ..., v_n\}$ has no perfect matching. But this contradicts the hypothesis.

Conversely, suppose that $|V(G)| \equiv n \pmod 2$ and for any vertex-set $S \subseteq V(G)$ with $|S| \geq n$, $o(G-S) \leq |S|-n$. If G is not n-critical, then there exist n vertices $v_1, v_2, ..., v_n$ such that $G-\{v_1, v_2, ..., v_n\}$ has no perfect matching. Using Tutte's theorem again, there exists a set $S' \subseteq V(G)-\{v_1, v_2, ..., v_n\}$ so that $o(G-\{v_1, v_2, ..., v_n\}-S') > |S'|$. Let $S = S' \cup \{v_1, v_2, ..., v_n\}$. Then $o(G-S) > |S'| = |S|-n$, a contradiction. □

There is another generalizations of n-extendability which consists of all graphs G satisfying the property that for any n-matching M and a set of m distinct vertices $u_1, u_2, ..., u_m$ of G, none of which is incident with any edge of M, there exists a perfect matching M* of G such that $M \subseteq M^*$ and $u_iu_j \notin M^*$ for $1 \leq i, j \leq m$ and $i \neq j$. This is called **(n,m)-extendability** and was studied by Liu and Yu [5]. This concept is stronger than n-extendability and is very helpful for studying the Cartesian products of n-extendable graphs.

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