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H-supermagic labelings for firecrackers, banana trees and flowers

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In memory of Mirka Miller

Abstract

A simple graph $G = (V, E)$ admits an H-covering if every edge in E is contained in a subgraph $H' = (V', E')$ of G which is isomorphic to H. In this case we say that G is H-supermagic if there is a bijection $f : V \cup E \to \{1, \ldots, |V| + |E|\}$ such that $f(V) = \{1, \ldots, |V|\}$ and

$\sum_{v\in V(H')} f(v) + \sum_{e\in E(H')} f(e)$ is constant over all subgraphs H' of G which are isomorphic to H. Extending results from [M. Roswitha and E.T. Baskoro, *Amer. Inst. Physics Conf. Proc.* 1450 (2012), 135-138], we show that the firecracker $F_{k,n}$ is $F_{2,n}$-supermagic, the banana tree $B_{k,n}$ is $B_{k-1,n}$-supermagic and the flower $\mathcal{F}_n$ is C_3-supermagic.

1 Introduction

The graphs considered in this paper are finite, undirected and simple. For a positive integer n we denote the set $\{1,\ldots,n\}$ by $[n]$, and for integers $a \leqslant b$, the set $\{a,\ldots,b\}$ is denoted by $[a,b]$. Let $V(G)$ and $E(G)$ be the set of vertices and edges of a graph G. A *graph labeling* is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labeling was first introduced by Rosa [8] in 1967. Since then there are various types of labeling that have been studied and developed (see [1]).

For a graph H, a graph G admits an *H-covering* if every edge of G belongs to at least one subgraph of G which is isomorphic to H. A graph $G=(V,E)$ which admits an H-covering is called *H-magic* if there exists a bijection $f : V \cup E \rightarrow [|V|+|E|]$ and a constant $f(H)$, which we call the *H-magic sum* of f, such that $\sum_{v\in V(H')} f(v) + \sum_{e\in E(H')} f(e) = f(H)$ for every subgraph $H' \subseteq G$ with $H' \cong H$. Additionaly, if $f(V) = [|V|]$ then we say that G is *H-supermagic.*

The concept of H-supermagic labeling was introduced by Gutiérrez and Lladó [2] in 2005, for H being a star or a path. In [4], Lladó and Moragas constructed cycle-supermagic labelings for some graphs. Furthermore, Maryati et al. [5] studied path-supermagic labelings while Ngurah et al. [7], Roswitha et al. [10] and Kojima [3] proved that some graphs have cycle-supermagic labelings. Some results for certain shackles and amalgamations of a connected graph have been proved by Maryati et al. [6]. Recently, Roswitha and Baskoro [9] established H-supermagic coverings for some trees.

Roswitha and Baskoro [9] show that for any integer k and even n, the firecracker graph $F_{k,n}$ is $F_{2,n}$-supermagic and the banana tree graph $B_{k,n}$ is $B_{(k-1),n}$-supermagic and left the remaining cases as open problems. In this paper, we solve these two problems. The result for banana trees is an immediate consequence of a theorem about amalgamations of graphs from [6] which we recall in Section 2. The result for firecrackers in Section 3 is obtained by a similar method. In addition, we prove in Section 4 that for odd n, the flower graph $\mathcal{F}_n$ is C_3-supermagic.

2 Amalgamations and banana trees

Let H be a graph with n vertices, say $V(H)=\{v_1,\ldots,v_n\}$ and m edges, say $E(H)=\{e_1,\ldots,e_m\}$. Take k copies of H denoted by $H^1,\ldots,H^k$ and let the vertex and edge sets be $V(H^i)=\{v_1^i,\ldots,v_n^i\}$ and $E(H^i)=\{e_1^i,\ldots,e_m^i\}$. Fix a vertex $v \in V(H)$, without loss of generality $v=v_n$, and form a graph, $G=A_k(H,v)$ by identifying all the vertices $v_n^1, \dots, v_n^k$ (and denoting the identified vertex by v_n). The following theorem was proved in [6].

Theorem 1 ([6])**.** *Let H be any graph, and let $v \in V(H)$. If $G = A_k(H, v)$ contains exactly k subgraphs isomorphic to H then G is H-supermagic with H-supermagic sums*

$$f(H) = \begin{cases} \frac{3(n+m)-1}{2} + \frac{k(n+m-1)^2}{2} & \text{if } (m+n-1)(k-1) \text{ is even,} \\ \frac{3(n+m)-2}{2} + \frac{k\left[(n+m-1)^2+1\right]}{2} & \text{if } (m+n-1)(k-1) \text{ is odd.} \end{cases}$$

For the convenience of the reader we provide an explicit description of the labeling.

Proof. The graph $A_k(H, v)$ has $k(n-1)+1$ vertices and km edges. We define the labeling

$$f : V(A_k(H, v)) \cup E(A_k(H, v)) \to [k(n+m-1)+1]$$

as follows.

Case 1 If $n+m$ is odd, we start with $f(v_n) = 1$. Then we use the labels $2, \dots, k(n-1)+1$ for the remaining vertices:

$$f(v_i^j) = \begin{cases} 1 + (i-1)k + j & \text{if } i \text{ is odd,} \\ ik + 2 - j & \text{if } i \text{ is even,} \end{cases} \qquad \text{for } i \in [n-1],\ j \in [k]. \tag{1}$$

Finally we use the labels $k(n-1)+2, \dots, k(n+m-1)+1$ for the edges:

$$f(e_i^j) = \begin{cases} 1 + (i+n-2)k + j & \text{if } i+n-1 \text{ is odd,} \\ (i+n-1)k + 2 - j & \text{if } i+n-1 \text{ is even,} \end{cases} \qquad \text{for } i \in [n-1],\ j \in [k]. \tag{2}$$

The sum of the labels used for H^j is independent of j:

$$\begin{aligned} f(v_n) + \sum_{i=1}^{n-1} f(v_i^j) + \sum_{i=1}^{m} f(e_i^j) &= 1 + \sum_{i=1,\, i \text{ odd}}^{n+m-1} [1 + (i-1)k + j] \\ &\quad + \sum_{i=1,\, i \text{ even}}^{n+m-1} [ik + 2 - j] \\ &= \frac{3(n+m)-1}{2} + \frac{k(n+m-1)^2}{2}. \end{aligned}$$

Case 2 If $n+m$ is even and k is odd, we start with $f(v_n) = 1$. Next we use the labels $2, \dots, 3k+1$ to label the vertices v_i^j for $i \in [3]$, $j \in [k]$ (assuming that $n \geqslant 4$, otherwise use the first edges in the obvious way):

$$f(v_1^j) = 1 + j$$

$$f(v_2^j) = \begin{cases} 3(k+1)/2 + j & \text{for } j \in [(k-1)/2], \\ (k+3)/2 + j & \text{for } j \in [(k+1)/2,\ k], \end{cases}$$

$$f(v_3^j) = \begin{cases} 3k + 2 - 2j & \text{for } j \in [(k-1)/2], \\ 4k + 2 - 2j & \text{for } j \in [(k+1)/2,\ k]. \end{cases}$$

Then we use the labels $3k+2,\dots,k(n-1)+1$ for the remaining vertices, applying (1) for $i \in [4, n-1]$. Finally, we use the labels $k(n-1)+2,\dots,k(n+m-1)+1$ for the edges, applying (2). The sum of the labels used for H^j is independent of j:

$$\begin{aligned} f(v_n) \quad &+ \quad \sum_{i=1}^{3} f(v_1^j) + \sum_{i=4}^{n-1} f(v_i^j) + \sum_{i=1}^{m} f(e_i^j) \\ &= \quad 1 + \frac{9(k+1)}{2} + \sum_{i=4,\, i \text{ odd}}^{n+m-1} [1+(i-1)k+j] + \sum_{i=4,\, i \text{ even}}^{n+m-1} [ik+2-j] \\ &= \quad \frac{3(n+m)-1}{2} + \frac{k(n+m-1)^2}{2}. \end{aligned}$$

Case 3 If $n+m$ is even and k is even, we start with $f(v_n) = k/2+1$. Next we use the labels $1,\dots,k/2,\ k/2+2,\dots,3k+1$ to label the vertices v_i^j for $i \in [3]$, $j \in [k]$ (assuming that $n \geqslant 4$, otherwise use the first edges in the obvious way):

$$f(v_1^j) = \begin{cases} j & \text{for } j \in [k/2], \\ j+1 & \text{for } j \in [k/2+1,\, k], \end{cases}$$

$$f(v_2^j) = \begin{cases} 3k/2+1+j & \text{for } j \in [k/2], \\ k/2+1+j & \text{for } j \in [k/2+1,\, k], \end{cases}$$

$$f(v_3^j) = \begin{cases} 3(k+1)-2j & \text{for } j \in [k/2], \\ 4k+2-2j & \text{for } j \in [k/2+1,\, k]. \end{cases}$$

Then we use the labels $3k+2,\dots,k(n-1)+1$ for the remaining vertices, applying (1) for $i = 4,\dots,n-1$. Finally, we use the labels $k(n-1)+2,\dots,k(n+m-1)+1$ for the edges, applying (2). The sum of the labels used for H^j is independent of j:

$$\begin{aligned} &f(v_n) + \sum_{i=1}^{3} f(v_1^j) + \sum_{i=4}^{n-1} f(v_i^j) + \sum_{i=1}^{m} f(e_i^j) \\ &= (k/2+1) + \frac{9k+8}{2} + \sum_{i=4,\, i \text{ odd}}^{n+m-1} [1+(i-1)k+j] + \sum_{i=4,\, i \text{ even}}^{n+m-1} [ik+2-j] \\ &\qquad\qquad = \frac{3(n+m)-2}{2} + \frac{k\left[(n+m-1)^2+1\right]}{2}. \quad \square \end{aligned}$$

Let H be the graph obtained by taking a star with n vertices and connecting an additional vertex v to exactly one leaf of the star. The *banana tree* $B_{k,n}$ is the graph $A_k(H, v)$.

Corollary 1. *For any integers k and $n \geqslant k+2$, the banana tree $B_{k,n}$ is $B_{1,n}$-supermagic.*

The condition $n \geqslant k+2$ is needed because otherwise $B_{k,n}$ contains more than k subgraphs isomorphic to $B_{1,n}$. We do not have this problems for $H = B_{\ell,n}$ with $\ell \geqslant 2$, and therefore we get the following result.

Corollary 2. *For any integers n, k and $\ell \in [2, k-1]$, the banana tree $B_{k,n}$ is $B_{\ell,n}$-supermagic. In particular, for $\ell = k-1$, this solves the open problem in [9].*

Remark 1. Note that the labeling strategy in the first case of the proof of Theorem 1 immediately gives the following result. Fix an induced subgraph H' of H, say induced by the last ℓ vertices, and form a graph, $G = A_k(H, H')$ by identifying the vertices $v_i^1, \ldots, v_i^k$ for $i = n-\ell+1, \ldots, n$. If $n - \ell + |E(H) \setminus E(H')|$ is even and G contains exactly k subgraphs isomorphic to H, then G is H-supermagic.

3 Attaching copies of a fixed graph to a path

Let G be a graph with n vertices, say $V(G) = \{v_1, \ldots, v_n\}$ and m edges, say $E(G) = \{e_1, \ldots, e_m\}$. Let P_k, $k \geqslant 2$, be a path with vertex set $V(P_k) = \{w_1, w_2, \ldots, w_k\}$ and edge set $E(P_k) = \{w_1w_2, \ldots, w_{k-1}w_k\}$. Take k copies of G denoted by $G^1, G^2, \ldots, G^k$ and let the vertex and edge sets be $V(G_i) = \{v_1^i, \ldots, v_n^i\}$ and $E(G^i) = \{e_1^i, \ldots, e_m^i\}$. Fix a vertex $v \in V(G)$, without loss of generality $v = v_n$, and attach the copies of G to the path such that the vertex $v_n^i \in V(G^i)$ is identified with the vertex w_i in P_k, $i = 1, 2, \ldots, k$. The resulting graph is denoted by $P_k(G, v)$.

Theorem 2. *Let G be graph with n vertices and m edges and let $k \geqslant 2$ be an integer. If $(n+m-1)(k-1)$ is even and $P_k(G,v)$ contains exactly $k-1$ subgraphs isomorphic to $P_2(G,v)$, then $P_k(G,v)$ is $P_2(G,v)$-supermagic with supermagic sum $(n+m)[(n+m+1)k+1] + \lceil k/2 \rceil$.*

Proof. The graph $P_k(G,v)$ has kn vertices and $(m+1)k-1$ edges. We define the labeling

$$f : V(P_k(G,v)) \cup E(P_k(G,v)) \to [(m+n+1)k-1]$$

as follows. For the path vertices, we use the first k labels $1, \ldots, k$:

$$f(w_i) = \begin{cases} (i+1)/2 & \text{if } i \text{ is odd,} \\ \lceil k/2 \rceil + i/2 & \text{if } i \text{ is even,} \end{cases} \qquad \text{for } i \in [k].$$

For the path edges, we use the the labels in $[(m+n)k+1, (m+n)k+(k-1)]$:

$$f(w_iw_{i+1}) = (n+m+1)k - i \qquad \text{for } i \in [k-1].$$

For labeling the remaining elements we distinguish two cases.

Case 1 If $n+m-1$ is even, we set

$$f\left(v_j^i\right) = \begin{cases} jk+i & \text{if } j \text{ is odd,} \\ (j+1)k+1-i & \text{if } j \text{ is even,} \end{cases} \qquad \text{for } j \in [n-1],\ i \in [k], \tag{3}$$

$$f\left(e_j^i\right) = \begin{cases} (n-1+j)k+i & \text{if } n-1+j \text{ is odd,} \\ (n+j)k+1-i & \text{if } n-1+j \text{ is even,} \end{cases} \qquad \text{for } j \in [m],\ i \in [k]. \tag{4}$$

Case 2 If $n+m-1$ is odd and k is odd, we set

$$f(v_1^i) = k+i$$
$$f(v_2^i) = \begin{cases} (5k+1)/2+i & \text{for } i \in [(k-1)/2], \\ (3k+1)/2+i & \text{for } i \in [(k+1)/2,\, k], \end{cases}$$
$$f(v_3^i) = \begin{cases} 4k+1-2i & \text{for } i \in [(k-1)/2], \\ 5k+1-2i & \text{for } i \in [(k+1)/2,\, k]. \end{cases}$$

As in the proof of Theorem 1, (3) and (4) are used for labeling the remaining vertices and edges.

Denoting the sum of the labels used for G^i by A_i, we obtain

$$A_i = \sum_{j=1}^{n-1} f(v_j^i) + \sum_{j=1}^{m} f(e_j^i) + f(w_i)$$
$$= \frac{(n+m-1)(n+m+1)k + (n+m-1)}{2} + \begin{cases} (i+1)/2 & \text{if } i \text{ is odd}, \\ \lceil k/2 \rceil + i/2 & \text{if } i \text{ is even}. \end{cases}$$

Finally, the sum of the labels of the subgraph isomorphic to $P_2(G,v)$ which is formed by G^i, G^{i+1} and the edge w_iw_{i+1} is independent of i:

$$A_i + A_{i+1} + f(w_iw_{i+1}) = (n+m)[(n+m+1)k+1] + \lceil k/2 \rceil. \qquad \square$$

Remark 2. We think that it might be possible that the parity assumption in Theorem 2 is not necessary, and we leave the case that both $n+m$ and k are even for future work.

Example 1. We illustrate the construction in Theorem 2 for $k=5$, $G=K_4^-$ (the graph obtained from a complete graph on 4 vertices by deleting one edge) and v being a vertex of degree 3 in G. We obtain the $P(K_4^-, v_4)$-supermagic labeling shown in Figure 1.

Corollary 3. *Let $G = K_{1,n-1}$ be a star with $n \geqslant 4$ vertices, and let v be a pendant vertex of G. The firecracker graph is $F_{k,n} = P_k(G,v)$. Since $|V(G)|+|E(G)| = 2n-1$ is odd, and there are exactly $k-1$ subgraphs isomorphic to $F_{2,n}$, the firecracker $F_{k,n}$ is $F_{2,n}$-supermagic with supermagic sum $(2n-1)(2nk+1)+\lceil k/2 \rceil$.*

4 C_3-Supermagic Labeling of the Flower Graph $\mathcal{F}_n$

A *flower* graph $\mathcal{F}_n$ is constructed from a wheel W_n by adding n vertices, each new vertex adjacent to one vertex on the cycle and the center of the wheel with vertex set $V = \{x_0\} \cup \{x_i : 1 \leqslant i \leqslant n\} \cup \{y_i : 1 \leqslant i \leqslant n\}$ and edge set $E = \{x_0x_i : 1 \leqslant i \leqslant n\} \cup \{x_0y_i : 1 \leqslant i \leqslant n\} \cup \{x_iy_i : 1 \leqslant i \leqslant n\} \cup \{x_ix_{i+1} : 1 \leqslant i \leqslant n\}$, where indices are interpreted modulo n in the obvious way.

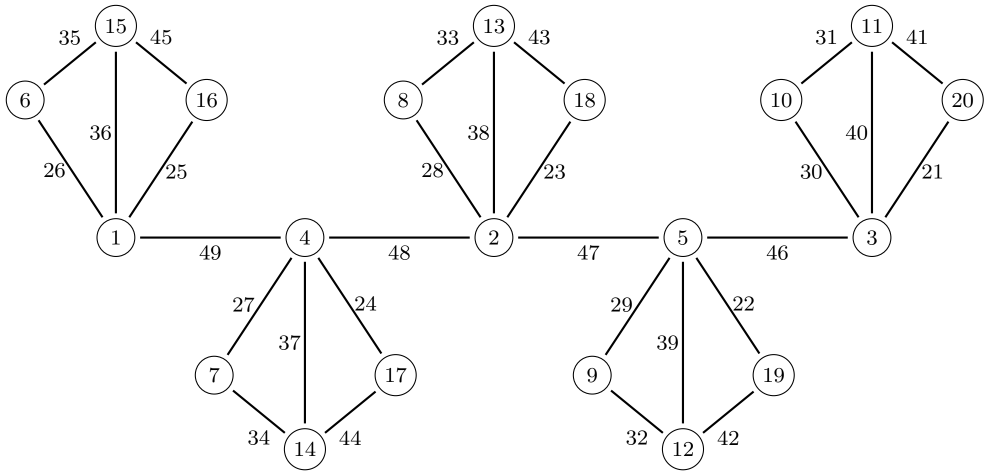

Figure 1: $P_2(K_4^-, v_4)$-supermagic labeling for $P_5(K_4^-, v_4)$.

We consider four permutations $\pi_1, \ldots, \pi_4$ of the set $[n]$, and define a total labeling of the flower graph $\mathcal{F}_n$ as follows.

$$\begin{aligned}
f(x_0) &= n+1, & \\
f(x_i) &= \pi_1(i) & \text{for } i \in [n],\\
f(y_i) &= \pi_2(i)+n+1 & \text{for } i \in [n],\\
f(x_0x_i) &= \pi_2(i)+5n+1 & \text{for } i \in [n],\\
f(x_0y_i) &= \pi_2(i)+4n+1 & \text{for } i \in [n],\\
f(x_iy_i) &= \begin{cases} \pi_3(i)+2n+1 & \text{odd } i \\ \pi_3(i)+3n+1 & \text{even } i \end{cases} & \text{for } i \in [n],\\
f(x_ix_{i+1}) &= \pi_4(i)+2n+1+(n+1)/2 & \text{for } i \in [n-1].
\end{aligned}$$

Lemma 1. *Define*

$$\varphi_1^i(\pi_1, \ldots, \pi_4) = \pi_1(i) + \pi_1(i+1) + \pi_2(i) + \pi_2(i+1) + \pi_4(i) + (n+1)/2 - 1,$$
$$\varphi_2^i(\pi_1, \ldots, \pi_4) = \pi_1(i) + 3\pi_2(i) + \pi_3(i) + \begin{cases} n & \textit{if } i \textit{ is even,} \\ 0 & \textit{if } i \textit{ is odd.} \end{cases}$$

If $\varphi_k^i(\pi_1, \ldots, \pi_4)$ is equal to a constant φ for all $i \in [n]$ and $k \in \{1, 2\}$, then the labeling given above is C_3-supermagic with supermagic sum $f(C_3) = 13n + 5 + \varphi$.

Proof. The flower graph F_n contains $2n$ subgraphs $H_1, \ldots, H_{2n}$ isomorphic to C_3. We distinguish two types of 3-cycles: (1) cycles induced by vertex sets $\{x_0, x_i, x_{i+1}\}$, and (2) cycles induced by vertex sets $\{x_0, y_i, y_i\}$.

Case 1 Cycle (x_0, x_i, x_{i+1}). The sum of the vertex labels is

$$f(x_0) + f(x_i) + f(x_{i+1}) = n + 1 + \pi_1(i) + \pi_1(i+1), \tag{5}$$

and the sum of the edge labels is

$$f(x_0x_i)+f(x_ix_{i+1})+f(x_0x_{i+1}) = \pi_2(i)+\pi_2(i+1)+\pi_4(i)+12n+3+(n+1)/2. \quad (6)$$

Taking the sum of (5) and (6), we have the supermagic sum

$$\begin{aligned} f(C_3) &= n + 1 + \pi_1(i) + \pi_1(i+1) + \pi_2(i) + \pi_2(i+1) + \pi_4(i) + 12n + 3 + (n+1)/2 \\ &= 13n + 5 + [\pi_1(i) + \pi_1(i+1) + \pi_2(i) + \pi_2(i+1) + \pi_4(i) - 1 + (n+1)/2] \\ &= 13n + 5 + \varphi_1^i(\pi_1, \dots, \pi_4) \\ &= 13n + 5 + \varphi. \end{aligned}$$

Case 2 Cycle (x_0, x_i, y_i). The sum of the vertex labels is

$$f(x_0) + f(x_i) + f(y_i) = 2n + 2 + \pi_1(i) + \pi_2(i), \quad (7)$$

and the sum of the edge labels is

$$f(x_0x_i)+f(x_iy_i)+f(x_0y_i) = \pi_2(i)+\pi_2(i)+\pi_3(i)+\begin{cases} 11n+3 & \text{if } i \text{ is odd,} \\ 12n+3 & \text{if } i \text{ is even.} \end{cases} \quad (8)$$

Taking the sum of (7) and (8) gives the supermagic sum

$$f(C_3) = 13n + 5 + \varphi_2^i(\pi_1, \dots, \pi_4) = 13n + 5 + \varphi. \qquad \square$$

In the following lemma we provide permutations $\pi_1, \dots, \pi_4$ which satisfy the condition in Lemma 1.

Lemma 2. *Define the permutations by*

$$\begin{aligned} \pi_1(i) &= i, \\ \pi_2(i) &= n + 1 - \begin{cases} (i+1)/2 & \textit{for odd } i, \\ i/2 + (n+1)/2 & \textit{for even } i, \end{cases} \\ \pi_3(i) &= \begin{cases} (i+1)/2 & \textit{for odd } i, \\ i/2 + (n+1)/2 & \textit{for even } i, \end{cases} \\ \pi_4(i) &= n + 1 - i. \end{aligned}$$

Then for every $i \in [n]$ we have $\varphi_1^i(\pi_1, \dots, \pi_4) = \varphi_2^i(\pi_1, \dots, \pi_4) = 3n + 2$.

Theorem 3. *For any odd integer n, the flower graph $\mathcal{F}_n$ is C_3-supermagic.*

Proof. The total labeling of $\mathcal{F}_n$ can be obtained by applying the permutations in Lemma 2 to the labeling construction. Using the value of φ in Lemma 2 and the supermagic the sum of the permutations in Lemma 2, we have the constant supermagic sum on flower graph $\mathcal{F}_n$

$$f(C_3) = 13n + 5 + \varphi = 13n + 5 + 3n + 2 = 16n + 7.$$

Hence, flower graph F_n is C_3-supermagic, for odd n. $\square$

Example 2. Using Lemma 2 for $n = 7$ we get the permutations $\pi_1 = (1, 2, 3, 4, 5, 6, 7)$, $\pi_2 = (7, 3, 6, 2, 5, 1, 4)$, $\pi_3 = (1, 5, 2, 6, 3, 7, 4)$ and $\pi_4 = (7, 6, 5, 4, 3, 2, 1)$. These permutations give the labeling for $\mathcal{F}_7$ shown in Figure 2.

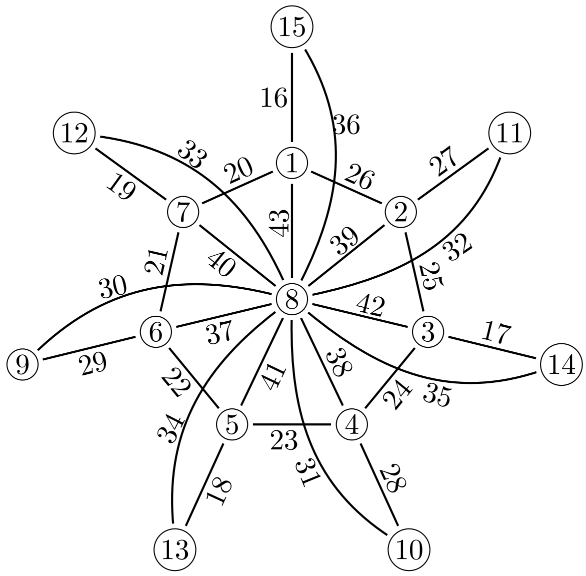

Figure 2: C_3-supermagic labeling of $\mathcal{F}_7$ with supermagic sum $f(C_3) = 119$.

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