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A characterization of the graphic split-off matroids

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Abstract

We consider the problem of determining precisely which graphs G have the property that the split-off operation, by every pair of edges, on the cycle matroid $M(G)$ yields a graphic matroid. This problem is solved by proving that there are exactly four minor-minimal graphs that do not have this property.

1 Introduction

The split-off operation for a binary matroid with respect to a pair of elements is defined as follows [9]: Let M be a binary matroid on a set E, and let A be a matrix that represents M over $GF(2)$. Suppose that $x, y \in E$, and the element $a \notin E$. Let A_{xy} be the matrix obtained from A by adjoining an extra column, with label a, which is the sum of the columns corresponding to x and y, and then deleting the two columns corresponding to x and y. Let M_{xy} be the vector matroid of the matrix A_{xy}. The transition from M to M_{xy} is called a split-off operation and the matroid M_{xy} is referred as the split-off matroid.

The split-off operation on a graphic matroid, in general, need not yield a graphic matroid. We characterize precisely those graphic matroids M having the property that for every pair $\{x, y\}$ of its elements, the matroid M_{xy} is graphic. This characterization is given in terms of the forbidden minors.

The following theorems are well known.

Theorem 1.1 [**10**] *A binary matroid is graphic if and only if it has no minor isomorphic to* F_7, F_7^*, $M^*(K_5)$, *or* $M^*(K_{3,3})$.

Theorem 1.2 [**1**] *Let* N *be a connected minor of a connected matroid* M *and suppose that* $e \in E(M) - E(N)$. *Then at least one of* $M \setminus e$ *and* M/e *is connected and has* N *as a minor.*

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The main result in this paper is the following theorem.

Theorem 1.3 *The split-off operation, by any pair of elements, on a graphic matroid yields a graphic matroid if and only if the matroid has no minor isomorphic to the cycle matroid of any of the following four graphs.*

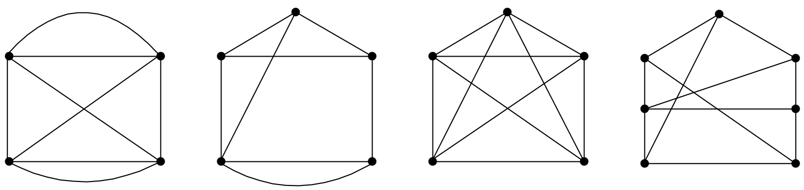

Figure 1

Notation. For the sake of convenience, we denote by $\mathcal{F}$ the class of matroids each of which is isomorphic to F_7, F_7^*, $M^*(K_5)$, or $M^*(K_{3,3})$.

For undefined notation and terminology in graphs and matroids, we refer the reader to [3, 5, 7].

2 The split-off operation and the minors

In this section, we state a few results concerning the split-off operation which play a vital role in the proof of the main result.

Proposition 2.1 [9] *Suppose $r(M)$ denotes the rank of a binary matroid M and x, y are distinct elements of M. Then (i) $r(M_{xy}) = r(M)$ if x and y are not in series; and (ii) $r(M_{xy}) = r(M) - 1$, otherwise.*

The following proposition provides relationships among the operations of deletion, contraction and split-off in binary matroids.

Proposition 2.2 [9] *Let M be a binary matroid on a set E, $x, y \in E$, and $a \notin E$. Let T be a subset of E such that $x, y \notin T$. Then*
(i) $(M \backslash T)_{xy} = (M_{xy}) \backslash T$;
(ii) $(M/T)_{xy} = (M_{xy})/T$; and
(iii) $(M_{xy}) \backslash \{a\} = M \backslash \{x, y\}$.

The proofs are straightforward. □

Theorem 2.3 *Let G be a graph and let x, y be edges of G. Suppose $M(G)_{xy}$ is not graphic. Then there is a graph G' in which no pair of edges is in series and $M(G')$ is a minor of $M(G)$ such that $M(G')_{xy} \in \mathcal{F}$ or $M(G')_{xy}/\{a\} \in \mathcal{F}$.*

Proof. If $M(G)_{xy}$ is not graphic then there are subsets X, Y such that $M(G)_{xy}/X \setminus Y \in \mathcal{F}$. If $a \notin X$ then we can take $G' = G/X \setminus Y$ and if $a \in X$ then can take

$G' = G/(X - \{a\}) \setminus Y$. It is not possible that $a \in Y$ for then $M(G)_{xy}$ would be graphic. □

Definition 2.4 Let G be a graph in which no pair of edges is in series and let $F \in \mathcal{F}$. We say that G is minimal with respect to F if there exist two edges x and y of G such that $M(G)_{xy} \cong F$ or $M(G)_{xy}/\{a\} \cong F$.

We deduce the following result as a corollary to Theorem 2.3.

Corollary 2.5 *Let $M = M(G)$ be a graphic matroid. Then the split-off operation, by any pair of elements, on M yields a graphic matroid if and only if G has no minor isomorphic to a minimal graph with respect to some matroid $F \in \mathcal{F}$.*

In the next theorem, we state some useful properties of minimal graphs.

Theorem 2.6 *Let G be a minimal graph with respect to F where $F \in \mathcal{F}$ and let x, y be two edges of G such that either $M(G)_{xy} \cong F$ or $M(G)_{xy}/\{a\} \cong F$. Then*

(i) G has no loops;

(ii) x and y are non-adjacent edges;

(iii) G is 2-connected;

(iv) if x_1 and x_2 are parallel edges of G then one of them must be either x or y; and

(v) if $M(G)_{xy}/\{a\} \cong F$, then G has at most one pair of parallel edges and there is no 4-cycle in G containing both x and y.

The proofs are straightforward. □

A matroid is said to be Euler if its ground set can be expressed as a union of disjoint circuits of the matroid (see [11]). If M is a binary Eulerian matroid and A is a matrix over $GF(2)$ that represents M then the sum of the columns of A is zero. Therefore it follows that a binary matroid M is Euler if and only if M_{xy} is Euler for every pair of elements x and y [9].

Proposition 2.7 *Let G be a graph and $x, y \in E(G)$. Then*

(i) $M(G)_{xy}$ is Euler if and only if G is Euler.

(ii) If x and y are nonadjacent and $M(G)_{xy}/\{a\}$ is Euler then either G is Euler or the end vertices of x and y are precisely the vertices of odd degree.

3 The split-off operation on graphic matroids

In this section, we investigate the minimal graphs corresponding to the four matroids F_7, F_7^*, $M^*(K_{3,3})$ and $M^*(K_5)$ (see Fig. 2) and use them to prove Theorem 1.3. In particular, the graphs to be examined have at most two more edges than each of the above matroids.

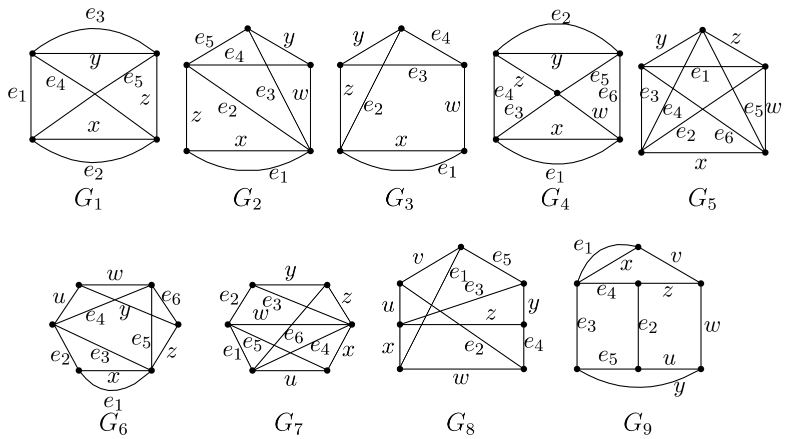

Figure 2

In the following lemma, we characterize minimal graphs corresponding to the Fano matroid F_7.

Lemma 3.1 *A graph is minimal with respect to the matroid F_7 if and only if it is isomorphic to one of the two graphs G_1 and G_2 of Figure 2.*

Proof. Clearly, neither of the graphs G_1 and G_2 of Figure 2 contains a pair of edges in series. Let A be a matrix over $GF(2)$ that represents the matroid $M(G_1)$ with respect to the basis $\{x, y, z\}$ (see Figure 2) and let A_{xy} be the matrix that represents $M(G_1)_{xy}$. Then

$$A = \begin{array}{c} \begin{array}{cccccccc} x & y & z & e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \end{array}; \text{ and } A_{xy} = \begin{array}{c} \begin{array}{ccccccc} a & z & e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \left[\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \end{array}.$$

The vector matroid of the matrix A_{xy} is isomorphic to F_7. Thus the graph G_1 is minimal with respect to F_7.

Now suppose B is a matrix over $GF(2)$ that represents the matroid $M(G_2)$ with respect to the basis $\{x, y, z, w\}$ (see Figure 2) and let B_{xy} be the matrix representing $M(G_2)_{xy}$. Then $B_{xy}/\{a\}$ represents a matroid that is isomorphic to F_7. Therefore, we conclude that the graph G_2 is minimal with respect to F_7.

Conversely, let G be a minimal graph with respect to F_7 and let x and y be edges of G such that either $M(G)_{xy} \cong F_7$ or $M(G)_{xy}/\{a\} \cong F_7$. Note that F_7 is a rank 3 matroid with 7 elements.

Case (i) Let $M(G)_{xy} \cong F_7$. Then $r(M(G)_{xy}) = 3$ and the ground set of $M(G)_{xy}$ has 7 elements. By Proposition 2.1(i), $r(M(G)) = 3$. Therefore G must have 4 vertices and 8 edges. Further, by Proposition 2.6, G is 2-connected and contains at most two pairs of parallel edges. Thus the simplification of G is the complete graph on four vertices. Moreover, G is obtained by adding two nonadjacent parallel edges to its simplification. Hence G is isomorphic to the graph G_1 of Figure 2.

Case (ii) Assume that $M(G)_{xy}/a \cong F_7$. Then in the light of Proposition 2.1(i) and Theorem 2.6(iii), G is 2-connected graph with 5 vertices and 9 edges. Now, by Theorem 2.6(v), x and y must be edges of a 5-cycle of G. Two of the possible edges from end-vertices of x to end-vertices of y can't be edges of G because a four cycle containing x and y is obtained. Thus the simplification of G is as given by G_2 deletes a parallel edge. Put the parallel edge back in the only possible place to get G_2. This completes the proof of the lemma. □

Lemma 3.2 *A graph G is minimal with respect to the matroid F_7^* if and only if it is isomorphic to the graph G_3.*

Proof. The graph G_3 has no pair of edges in series. Let A be a matrix over $GF(2)$ that represents $M(G_3)$ with respect to the basis $\{x, y, z, w\}$ (see Figure 2). Then the vector matroid of the matrix A_{xy} is isomorphic to F_7^*. Thus, we conclude that the graph G_3 is minimal with respect to F_7^*.

Conversely, let G be a minimal graph with respect to F_7^* and let x and y be edges of G such that either $M(G)_{xy} \cong F_7^*$ or $M(G)_{xy}/\{a\} \cong F_7^*$. We note that F_7^* is a rank-4 matroid with seven elements in which every circuit of it has four elements.

Case (i) Suppose that $M(G)_{xy} \cong F_7^*$. Then, by Proposition 2.1(i) and Theorem 2.6, G must have 5 vertices, 8 edges and degree sequence (4, 3, 3, 3, 3). If G is simple then it should be isomorphic to $G_5 \setminus \{e_2, e_6\}$ (see [3], p. 217). However, every pair of non-adjacent edges in this graph is contained in a 4-circuit which will yield a 3-circuit of F_7^*, a contradiction. Hence G cannot be simple. Suppose G is a multigraph. Then it must be obtained from a simple graph with 5 vertices and 7 edges by putting an edge in parallel. There is only one simple graph of this type (see [3], p. 217). Indeed, this graph is isomorphic to $G_5 \setminus \{e_2, e_4, e_6\}$ and contains two edges in series. The graph G is obtained from this graph by putting an edge parallel to one of the two edges which are in series. Consequently, G is isomorphic to the graph G_3 of Figure 2.

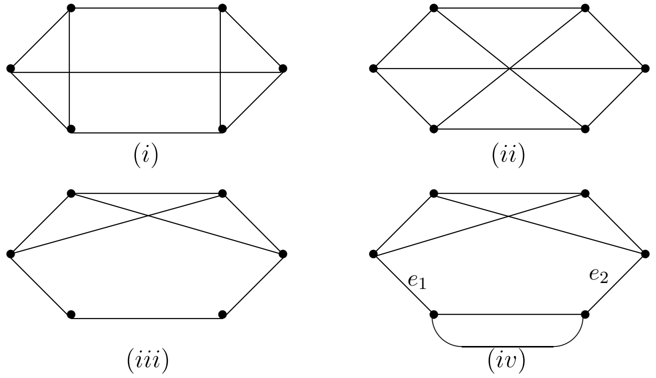

Figure 3

Case (ii) Let $M(G)_{xy}/\{a\} \cong F_7^*$. By Proposition 2.1(i) and Theorem 2.6(iii), the degree sequence for G is $(3,3,3,3,3,3)$. If G is simple then it must be one of the two graphs (i) and (ii) of Figure 3 (see [3], p. 222). But every pair of non-adjacent edges in each of the two graphs is contained in either a 4-circuit or a 5-circuit which will give rise to a 2-circuit and a 3-circuit of $M(G)_{xy}/\{a\}$, but this is not impossible. Hence G cannot be simple. If G is a multigraph then, by Theorem 2.6(iv), it has at most one pair of parallel edges. Thus, G can be obtained from a simple graph, say G', with 6 vertices and 8 edges, by adding an edge parallel to the edge which has each of the end vertices of degree two. The graph G' has degree sequence $(3,3,3,3,2,2)$ and the vertices of degree 2 must be adjacent. Indeed, the graph G' is isomorphic to the graph (iii) of Figure 3 (see [3], p.221). The corresponding graph G is isomorphic to the graph (iv) of Figure 3. However, the edges e_1 and e_2 of this graph are in series and hence graph (iv) is not minimal with respect to F_7^*. This completes the proof of the lemma. $\square$.

The following lemma characterizes minimal graphs corresponding to the matroid $M^*(K_{3,3})$.

Lemma 3.3 *A graph is minimal with respect to the matroid $M^*(K_{3,3})$ if and only if it is isomorphic to one of the four graphs G_4, G_5, G_6 and G_7 of Figure 2.*

Proof. Neither of the graphs G_4, G_5, G_6 and G_7 of Figure 2 contains pairs of edges in series. Further, each of the matroids $M(G_4)_{xy}$, $M(G_5)_{xy}$, $M(G_6)_{xy}/\{a\}$, and $M(G_7)_{xy}/\{a\}$ is isomorphic to $M^*(K_{3,3})$. Therefore, each of the graphs G_4, G_5, G_6 and G_7 is minimal with respect to $M^*(K_{3,3})$.

Now, suppose G is a minimal graph with respect to $M^*(K_{3,3})$ and let x and y be the edges of G with the property that $M(G)_{xy} \cong M^*(K_{3,3})$ or $M(G)_{xy}/\{a\} \cong M^*(K_{3,3})$. $M^*(K_{3,3})$ is a matroid with 9 elements and its rank is 4.

Case (i) $M(G)_{xy} \cong M^*(K_{3,3})$. In the light of Proposition 2.1(i) and Theorem 2.6, G has 5 vertices, 10 edges and no pair of edges of G is in series. Since $M^*(K_{3,3})$ is Euler, by Proposition 2.7(i), G must be Euler. If G is simple, then it is isomorphic to G_5 (see [3], p.217). Suppose G is a multigraph and has one pair of parallel edges. Then it must be obtained from K_5 by deleting one of its edges and adding a parallel edge to an edge having each end vertex of degree four. But then such a graph is not Euler. Hence G cannot have just one pair of parallel edges. If G has two pairs of parallel edges then G is obtained from K_5 by deleting two nonadjacent edges and then putting two parallel edges in such a way that the resulting graph is Euler. The graph G must be isomorphic to the graph G_4 of Figure 2.

Case (ii) Assume that $M(G)_{xy}/\{a\} \cong M^*(K_{3,3})$. Then, by Proposition 2.1(i) and Theorem 2.6, G must be a 2-connected graph with 6 vertices and 11 edges. Further, by Proposition 2.7(ii), G is Euler or each of the end vertices of x and y are precisely of odd degree. Figure 4 shows all 2-connected simple graphs on 6 vertices and 11 edges ([3], p. 223).

Every pair of non-adjacent edges in each of the graphs (iii) and (iv) of Figure 4 is contained in a 4-cycle and therefore, by Theorem 2.6(v), neither of these graphs is minimal. The graph (ii) of Figure 4 is nothing but the graph G_7 of Figure 2. Each

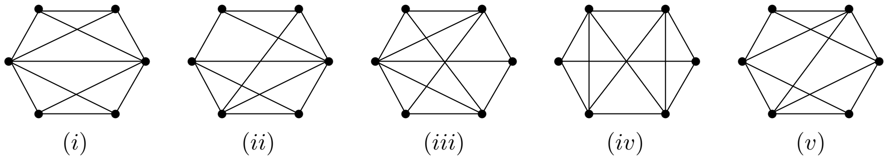

Figure 4

of the remaining graphs of Figure 4 does not satisfy either of the two properties stated in Proposition 2.7(ii). Hence neither of these graphs is minimal with respect to $M^*(K_{3,3})$.

Now suppose that the graph G is a multigraph. Then, by Theorem 2.6(iv), it contains at most one pair of parallel edges and, by minimality, it has no pair of edges in series. Thus G is obtainable from a connected simple graph with 6 vertices and 10 edges, by putting a parallel edge at a suitable place. The simple connected graphs each with 6 vertices and 10 edges are shown in Figure 5 ([3], p. 223).

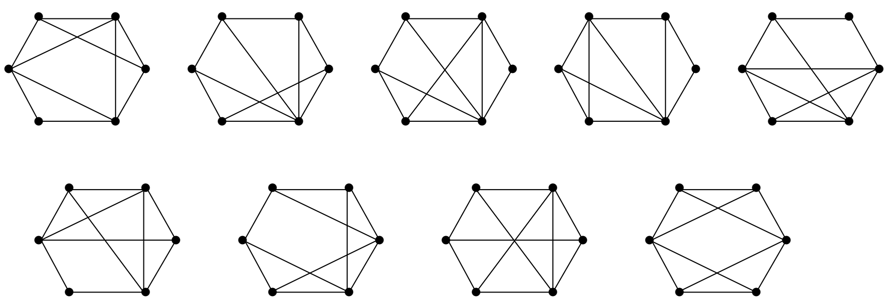

Figure 5

Consider the first graph of the first row of Figure 5. It contains a vertex of degree two. If a parallel edge is to be put up in this graph to obtain the graph G the edge must be put up parallel to an edge which has one end vertex of degree two. There are two ways to put a parallel edge in this graph. In fact, the two ways are symmetric and give rise to two graphs each of which is isomorphic to the graph G_6 of Figure 2. In each of the remaining graphs of Figure 5 if we put a parallel edge in all possible ways then each of the resulting graphs does not satisfy either of the two conditions stated in Proposition 2.7(ii). Thus none of these graphs is minimal with respect to $M^*(K_{3,3})$. This completes the proof. □

The following lemma characterizes the minimal graphs corresponding to the matroid $M^*(K_5)$.

Lemma 3.4 *A graph is minimal with respect to $M^*(K_5)$ if and only if it is isomorphic to G_8 or G_9.*

Proof. Neither of the graphs G_8 and G_9 of Figure 2 has a pair of edges in series. Moreover, each of the matroids $M(G_8)_{xy}$ and $M(G_9)_{xy}$ is isomorphic to $M^*(K_5)$. Thus, the graphs G_8 and G_9 are minimal with respect to $M^*(K_5)$.

Conversely, suppose that G is a minimal graph with respect to $M^*(K_5)$ and let x, y be a pair of edges in G such that either $M(G)_{xy} \cong M^*(K_5)$ or $M(G)_{xy}/\{a\} \cong M^*(K_5)$. We recall that $M^*(K_5)$ is a bipartite matroid with 10 elements and rank 6. In fact, it has 5 circuits of size 4 and 10 circuits of size 6.

Case (i) Suppose $M(G)_{xy} \cong M^*(K_5)$. Then, by Proposition 2.1(i), and Theorem 2.6, G has degree sequence $(4, 3, 3, 3, 3, 3, 3)$. From the nature of circuits of $M^*(K_5)$ and the definition of $M(G)_{xy}$, it follows that G cannot have (i) two or more edge-disjoint triangles and (ii) two or more pairs of parallel edges. Suppose G is simple. The non-isomorphic simple graphs each of which has degree sequence $(4, 3, 3, 3, 3, 3, 3)$ can be constructed from the simple graphs with degree sequence $(3, 3, 2, 2, 2, 2)$ by adding a vertex adjacent to vertices of degree two. There are precisely four non-isomorphic simple graphs with degree sequence $(3, 3, 2, 2, 2, 2)$ ([3], p. 220). The non-isomorphic simple graphs each with degree sequence $(4, 3, 3, 3, 3, 3, 3)$ are shown in Figure 6. However, each of the graphs (i), (ii) and (iii) of Figure 6 has two or more triangles and therefore none of these graphs is minimal with respect to $M^*(K_5)$. The graph (iv) of Figure 6 is the graph G_8 of Figure 2.

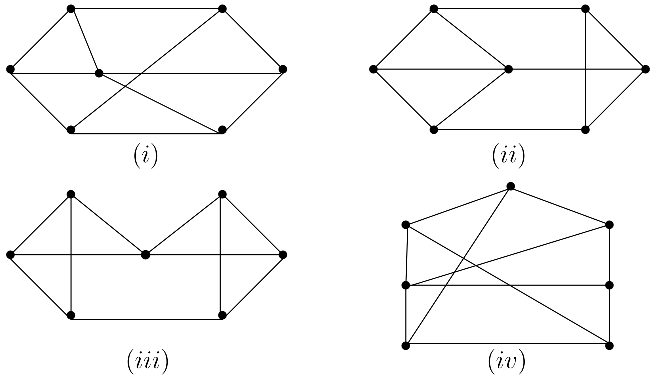

Figure 6

If G is not simple then, as noted above, it has only one pair of parallel edges. Thus G can be obtained from a simple graph with degree sequence $(4, 3, 3, 3, 3, 2, 2)$ or $(3, 3, 3, 3, 3, 3, 2)$ by adding an edge in parallel. However, any graph obtained by this way from a simple graph with degree sequence $(4, 3, 3, 3, 3, 2, 2)$ contains a pair of edges in series. Thus G cannot be obtained from the first type of graphs. Now all non-isomorphic simple graphs with degree sequence $(3, 3, 3, 3, 3, 3, 2)$ can be obtained from the non-isomorphic simple graphs each with degree sequence $(3, 3, 2, 2, 2, 2)$ (see [3], pp. 220–221). There are five simple graphs each with degree sequence $(3, 3, 3, 3, 3, 3, 2)$. The multigraphs obtained from each of the five graphs, by adding a parallel edge to an edge having a end vertex of degree 2, are shown in Figure 7.

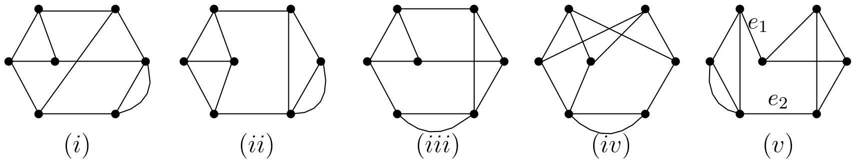

Figure 7

Now, each of the graphs (ii), (iii) and (v) of Figure 7 contains a pair of edge-disjoint triangles and hence none of them is minimal. In graph (iv) of Figure 7, one of the two edges x and y, say x must be in a 2-circuit and the edge y being non-adjacent with x must be contained in a 4-circuit which is disjoint from the 2-circuit. But then $M(G)_{xy}$ will contain a circuit whose size is different from 4 and 6, which is impossible. Thus G must be isomorphic to graph (i) of Figure 7 which is nothing but the graph G_9 of Figure 2.

Case (ii) Assume that $M(G)_{xy}/\{a\} \cong M^*(K_5)$. In view of Proposition 2.1 and Theorem 2.6, the degree sequence of G is (3, 3, 3, 3, 3, 3, 3, 3). Suppose G is simple. Then G is obtainable from a simple graph having degree sequence $(2, 2, 2, 2, 2, 2)$, $(3, 2, 2, 2, 2, 1)$, $(3, 3, 2, 2, 1, 1)$ or $(3, 3, 3, 1, 1, 1)$ by adding two more vertices adjacent to vertices of degree two and one. There are 11 non-isomorphic graphs each of which has one of the above degree sequence (see [3], p. 221, 222). Figure 8 shows all non-isomorphic 2-connected simple graphs each of which has degree sequence $(3, 3, 3, 3, 3, 3, 3, 3)$ and no pair of edges in series.

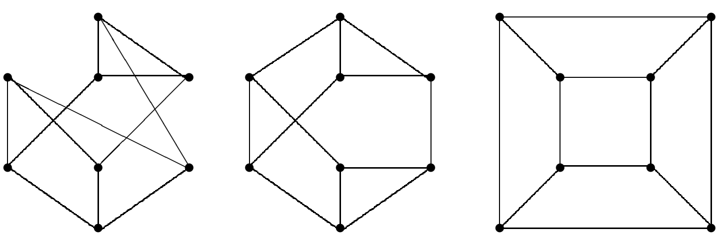

Figure 8

The matroid $M^*(K_5)$ has only even sized circuits and contains 5 circuits each of size four. Therefore the nonadjacent edges x and y satisfy the following properties.

i) If G contains a triangle then precisely one of x and y must be an edge of the triangle.

ii) G has no cycle of odd size which contains both x and y.

iii) The total number of 4-cycles containing neither x nor y and 6-cycles containing both x and y should not exceed 5.

iv) By Proposition 2.6(v), G has no 4-cycle which contains both x and y.

Now one can check that neither of the three graphs of Figure 8 contains a pair of nonadjacent edges x and y which satisfy the above properties. Thus there cannot be a simple graph G with the condition $M(G)_{xy}/\{a\} \cong M^*(K_5)$.

Suppose G is multigraph. Then, by Theorem 2.6(v), it has at most one pair of parallel edges. Accordingly, G is obtained from a connected simple graph, say H, with 8 vertices and 11 edges by adding an edge in parallel. Further, the graph H must have two adjacent vertices each of which has degree two and the edge to be added must be parallel to the edge joining these two vertices. But the two edges of G adjacent to the pair of parallel edges are in series. We conclude that there is no graph G such that $M(G)_{xy}/\{a\} \cong M^*(K_5)$. □

Now we use Lemmas 3.1, 3.2, 3.3 and 3.4 to prove Theorem 1.3.

Proof of Theorem 1.3. Let $M = M(G)$ be a graphic matroid. On combining Corollary 2.5 and Lemmas 3.1, 3.2, 3.3, and 3.4, it follow that $M(G)_{xy}$ is graphic for every pair $\{x, y\}$ of edges of G if and only if $M(G)$ has no minor isomorphic to any of the matroids $M(G_i)$, $i = 1, 2, \ldots, 9$ where the graphs $G_i s$ are shown in Figure 2. However, we have

$$\begin{aligned} M(G_3) &\cong M(G_2) \setminus e_2 \\ &\cong M(G_4) \setminus \{e_2, w\} \\ &\cong M(G_6)/e_2 \setminus \{e_3, e_5\} \\ &\cong M(G_7)/e_1 \setminus \{w, e_4\} \\ &\cong M(G_9)/\{v, z\} \setminus e_1. \end{aligned}$$

Thus, $M(G)_{xy}$ is graphic if and only if $M(G)$ has no minor isomorphic to any of the matroids $M(G_1)$, $M(G_3)$, $M(G_5)$ and $M(G_8)$. But the graphs G_1, G_3, G_5 and G_8 are precisely the graphs given in the statement of the theorem. □

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References

[1] T. H. Brylawski, A decomposition for combinatorial geometries, *Trans. Amer. Math. Soc.* 171 (1972), 235–282.

[2] A. Frank, Augmenting graphs to meet edge-connectivity requirements, *SIAM J. of Discrete Math.*, 5 (1) (1992), 22–53.

[3] F. Harary, *Graph theory*, Narosa Publishing House, New Delhi (1996).

[4] L. Lovasz, *Combinatorial problems and exercises*, North Holland, Amsterdam (1979).

[5] J. G. Oxley, *Matroid Theory*, Oxford University Press, New York (1992).

[6] T. T. Raghunathan, M. M. Shikare and B. N. Waphare, Splitting in a binary matroid, *Discrete Math.*, 184 (1998), 267–271.

[7] A. Recski, *Matroid theory and its applications*, Springer Verlag, Berlin (1989).

[8] M. M. Shikare and G. Azadi, Determination of the bases of a splitting matroid, *Europ. J. Combin.* 24 (2003), 45–52.

[9] M. M. Shikare, K. V. Dalvi and S. B. Dhotre, Splitting off operation for binary matroids and its applications, *Graphs and Combinatorics* (to appear).

[10] M. M. Shikare and B. N. Waphare, Excluded-Minors for the class of graphic splitting matroids, *Ars Combinatoria* 97 (2010), 111–127.

[11] W. T. Tutte, Lectures on matroids, *J. Res. Nat. Bur. Standards Sect.* 69B (1965), 1–47.

[12] D. J. A. Welsh, Euler and bipartite matroids, *J. Combin. Theory* 6 (1969), 375–377.

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