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The negative decision number in graphs

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Abstract

A *bad function* is a function $f : V(G) \to \{-1, 1\}$ satisfying $\sum_{v \in N(v)} f(v) \leq 1$ for every $v \in V(G)$, where $N(v) = \{u \in V(G) \mid uv \in E(G)\}$. The maximum of the values of $\sum_{v \in V(G)} f(v)$, taken over all bad functions f, is called the *negative decision number* and is denoted by $\beta_D(G)$. In this paper, several sharp upper bounds of this number for general graphs are presented.

1 Introduction

All graphs considered in this paper are finite, simple, and undirected. For a general reference on graph theory, the reader is directed to [1].

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For a vertex $v \in V(G)$, the open neighbourhood of v is $N(v) = \{u \in V(G) \mid uv \in E(G)\}$, and the closed neighbourhood of v is $N[v] = N(v) \cup \{v\}$. For a subset $S \subseteq V(G)$, $\deg_S(v)$ denotes the number of vertices in S adjacent to v. In particular, $\deg_{V(G)}(v) = \deg(v)$, the degree of v in G. For disjoint subsets S and T of vertices, we use $E(S, T)$ for the set of edges between S and T, and let $e(S, T) = |E(S, T)|$. The subgraph of G induced by S is denoted by $G[S]$. For a fixed $k \geq 2$, we call $K_{1,k}$ a *k-star.* A *k^*-star* is the graph obtained from a k-star by subdividing each edge once. We call the vertex with degree k in the original k-star the *central* vertex. See Figure 1 for a copy of 5^*-star. Let $x : V(G) \to \mathbb{R}$ be a real-valued function. We write $x(S)$ for $\sum_{v \in S} x(v)$

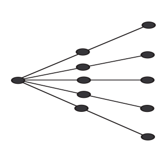

Figure 1: A copy of 5^*-star

for $S \subseteq V(G)$.

Domination in graphs is well studied in graph theory. The literature on this subject has been detailed in the two books [7, 8]. The signed domination has been investigated in, for instance, [2, 3, 4, 5, 6, 9, 10, 11, 12, 14].

A *signed total dominating function* is a function $f : V(G) \to \{-1, 1\}$ satisfying $\sum_{v \in N(v)} f(v) \geq 1$ for every $v \in V(G)$. The minimum of the values of $\sum_{v \in V(G)} f(v)$, taken over all signed total dominating functions f, is called the *signed total domination number* and is denoted by $\gamma_t^S(G)$.

In this paper, we initiate the study of a new graph parameter by changing "$\geq$" to "$\leq$" in the definition of signed total domination number.

A function $f : V(G) \to \{-1, 1\}$ is called a *bad function* (**BF**) of G if $f(N(v)) \leq 1$ for every $v \in V(G)$. The maximum of the values of $f(V(G))$, taken over all bad functions f, is called the *negative decision number* and is denoted by $\beta_D(G)$.

The motivation for studying this new parameter may be varied from a modelling perspective. For instance, by assigning the values -1 or 1 to the vertices of a graph one can model networks of people in which global decisions must be made (e.g. positive or negative responses). In certain circumstances, a positive decision can be made only if there are significantly more people voting for than those voting against. We assume that each individual has one vote, and each has an initial opinion. We assign 1 to vertices (individuals) which have a positive opinion and -1 to vertices which have a negative opinion. A voter votes 'good' if there are *two* more vertices in its open neighborhood with positive opinion than with negative opinion, otherwise the vote is 'bad'. We seek an assignment of opinions that guarantee a unanimous decision; namely, for which every vertex votes 'bad'. Such an assignment of opinions is called a uniformly negative assignment. Among all uniformly negative assignments of opinions, we are particularly interested in the minimum number of vertices (individuals) which have a negative opinion. The negative decision number is the maximum possible sum of all opinions, 1 for a positive opinion and -1 for a negative opinion, in a uniformly negative assignment of opinions. The negative decision number corresponds the minimum number of individuals who can have negative opinions and in doing so force every individual to vote bad.

Throughout this paper, if f is a BF of G, then we let P and Q denote the sets of those vertices of G which are assigned (under f) the values 1 and -1, respectively, and we let $p = |P|$ and $q = |Q|$. Therefore, $f(V(G)) = p - q$.

We establish upper bounds of $\beta_D(G)$ for a bipartite graph and a general graph in terms of their orders and we characterize the graphs attaining these bounds. We present a sharp upper bound of $\beta_D(G)$ for a general graph in terms of its order and size. We also establish a sharp upper bound of $\beta_D(G)$ for a k-regular graph in terms of its order. Exact values of $\beta_D(G)$ for some familiar graphs such as cycles, paths, cliques and bicliques are found.

2 Upper bounds of $\beta_D(G)$

2.1 General graphs

In this subsection, we first present an upper bound of $\beta_D(G)$ for a general graph with minimum degree at least 2 in terms of its order and we characterize the graphs attaining this bound.

We define a family $\mathcal{F}$ of graphs as follows. For $k \geq 2$, let F_k be the set of the graphs G obtained from the disjoint union of k k-stars by adding all possible edges between their central vertices $a_1, a_2, \ldots, a_k$, and adding edges among vertices in $V' = V(G)\backslash\{a_1, a_2, \ldots, a_k\}$ so that each vertex in the subgraph $G[V']$ has degree between 1 and 2. See Figure 2 for two graphs in F_4. In particular, when k is even, we use J_k to denote all graphs in F_k satisfying that the induced subgraph $G[V']$ is a perfect matching. Let $\mathcal{F} = \bigcup_{k\geq 2} G_k$.

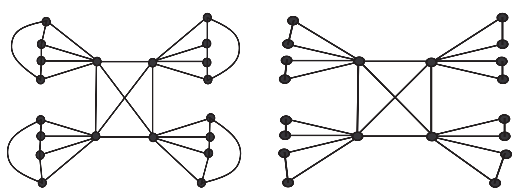

Figure 2: Two graphs in F_4.

Theorem 1. *If G is a graph of order n with minimum degree at least 2, then*

$$\beta_D(G) \leq n + 1 - \sqrt{4n+1}.$$

The equality holds if and only if $G \in \mathcal{F}$.

Proof. Let f be a BF such that $\beta_D(G) = f(V(G))$. Then $\beta_D(G) = |P| - |Q| = n - 2q$. Notice that every vertex in P must be joined to at least one vertex in Q. By the pigeonhole principle, there exists a vertex v in Q joined to at least $|P|/|Q| = (n-q)/q$ vertices in P. Thus,

$$\begin{aligned} 1 &\geq f(N(v)) \\ &\geq -(|Q|-1) + (n-q)/q \\ &\geq -q + 1 + (n-q)/q. \end{aligned}$$

i.e.,

$$q^2 + q - n \geq 0.$$

Solving the above inequality for q, we obtain that

$$q \geq \frac{1}{2}(-1 + \sqrt{4n+1}).$$

Therefore, $\beta_D(G) = n - 2q \le n + 1 - \sqrt{4n+1}$.

If $\beta_D(G) = n - 2q = n + 1 - \sqrt{4n+1}$, then $n = q^2 + q$ and $p = q^2$. Furthermore, each vertex of P is joined to a vertex of Q and has degree between 2 and 3, while each vertex of Q is joined to all other $q-1$ vertices of Q and to q vertices of P. It follows that $G \in \mathcal{F}$.

If $G \in \mathcal{F}$, then $G \in F_k$ for some $k \ge 2$. Thus, G has order $n = k^2 + k$, and so $k = \frac{1}{2}(-1 + \sqrt{4n+1})$. Assigning -1 to each of the k central vertices, and 1 to all other vertices, we define a BF f of G satisfying $f(V(G)) = k^2 - k = n+1-\sqrt{4n+1}$. Thus, $\beta_D(G) \ge n + 1 - \sqrt{4n+1}$. Consequently, $\beta_D(G) = n + 1 - \sqrt{4n+1}$. □

We remark that Theorem 1 is not true for the graphs G with minimum degree 1. To see this, we define a family $\mathcal{F}'$ of graphs as follows. For $k \ge 2$, let F'_k be the graph obtained from the disjoint union of k $(k+1)$-stars and a copy of K_k by adding all possible edges between central vertex b_j of $(k+1)$-star and the vertices of K_k for each $1 \le j \le k$. See Figure 3 for a copy of F'_2. Clearly, F'_k is a graph

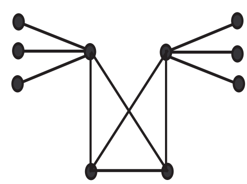

Figure 3: A copy of F'_2 with $\beta_D(F'_2) = 6$.

of order $n = k^2 + 3k$ with minimum degree 1. Assigning the value -1 to each of the k vertices of K_k, and $+1$ to all other vertices, we define a BF f of G satisfying $f(V(G)) = k^2 + k > n + 1 - \sqrt{4n+1}$. Thus, $\beta_D(G) > n + 1 - \sqrt{4n+1}$.

Next we establish an upper bound of $\beta_D(G)$ for a general graph with minimum degree at least 2 in terms of its order and size.

Theorem 2. *If G is a graph of order n and size m with minimum degree at least 2, then*

$$\beta_D(G) \le \frac{1}{5}(4m - 3n),$$

and this bound is sharp.

Proof. Let f be a BF such that $\beta_D(G) = f(V(G))$. Then $\beta_D(G) = |P| - |Q| = n - 2q$. Let $P_0 = \{v \in P | \deg_P(v) = 0\}$ and $|P_0| = t$ $(0 \le t \le n - q)$.

Note that $\delta(G) \ge 2$, so we have

$$e(P, Q) \ge p + t \ge n - q,$$

and

$$|E(G[P])| \ge (n - q - t)/2.$$

For each vertex v of Q, $\deg_Q(v) \geq \deg_P(v) - 1$. Hence,

$$n - q \leq e(P,Q) = \sum_{v \in Q} \deg_P(v) \leq \sum_{v \in Q} \left(\deg_Q(v) + 1\right).$$

i.e.,

$$n - q \leq 2\,|E(G[Q])| + q.$$

So,

$$|E(G[Q])| \geq (n - 2q)/2.$$

Thus, the total number of edges in G is

$$\begin{aligned} m &= |E(G[Q])| + e(P,Q) + |E(G[P])| \\ &\geq (n-2q)/2 + (n-q+t) + (n-q-t)/2 \\ &\geq (n-2q)/2 + 3(n-q)/2. \end{aligned}$$

Solving the above inequality for q, we obtain that

$$q \geq \frac{2}{5}(2n - m).$$

Thus,

$$\beta_D(G) = n - 2q \leq \frac{1}{5}(4m - 3n).$$

To see this bound is sharp, let k be an even positive integer and let $G \in J_k \subset \mathcal{F}$. Thus, G has order $n = k^2 + k$ and size $m = \frac{3}{2}k^2 + \frac{1}{2}k(k-1) = 2k^2 - k/2$. As seen in the proof of Theorem 1, $\beta_D(G) = n + 1 - \sqrt{4n+1} = k^2 - k = \frac{1}{5}(4m - 3n)$. □

2.2 Bipartite graphs

In this subsection, we present an upper bound of $\beta_D(G)$ for a general bipartite graph and we characterize the graphs attaining this bound. We define a family $\mathcal{H}$ of bipartite graphs as follows.

For $k \geq 2$, let H_k be the bipartite graph obtained from the disjoint union of $2k$ $(k+1)^*$-stars with centres $\{x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_k\}$ by adding all edges of the type $x_i y_j$, $1 \leq i \leq j \leq k$. Let $\mathcal{H} = \{H_k | k \geq 2\}$.

Theorem 3. *If G is a bipartite graph of order n, then*

$$\beta_D(G) \leq n + 3 - \sqrt{4n+9}.$$

The equality holds if and only if $G \in \mathcal{H}$.

Proof. Let f be a BF of G such that $\beta_D(G) = f(V(G))$. Let X and Y be the partite sets of G. Further, let X^+ and X^- be the sets of vertices in X that are assigned the value $+1$ and -1 (under f), respectively. Let $X_1^+ \subseteq X^+$ be the vertices which are joined to none of Y^-. Hence, each vertex in X_1^+ is joined to exactly one vertex in Y^+ which is also joined to some vertex(vertices) in X^-. Let Y^+, Y^- and Y_1^+ be defined analogously. Then $P = X^+ \cup Y^+$ and $Q = X^- \cup Y^-$. For convenience, let $|X^+| = k, |X_1^+| = k_1, |X^-| = s, |Y^+| = l, |Y_1^+| = l_1$ and $|Y^-| = t$. Hence, $\beta_D(G) = k + l - s - t = n - 2(s + t)$.

Every vertex in $X^+ \backslash X_1^+$ must be joined to at least one vertex in Y^-. Therefore, by the pigeonhole principle, there is a vertex v in Y^- joined to at least $|X^+ \backslash X_1^+|/|Y^-| = (k - k_1)/t$ vertices in $X^+ \backslash X_1^+$. Hence,

$$1 \geq f\left(N(v)\right) \geq -|X^-| + |X^+ \backslash X_1^+|/|Y^-| = -s + (k - k_1)/t.$$

i.e.,

$$t(s + 1) \geq k - k_1. \tag{1}$$

By a similar argument, one may show that

$$s(t + 1) \geq l - l_1. \tag{2}$$

We observe that

$$k - k_1 \geq l_1, \tag{3}$$

as otherwise there is a vertex $u \in Y_1^+$ joined to at least two vertices in X^+, which is a contradiction.

Similarly,

$$l - l_1 \geq k_1. \tag{4}$$

Adding (3) and (4), we obtain that

$$k + l \geq 2(k_1 + l_1). \tag{5}$$

By (1), (2) and (5), we have that

$$2st + s + t \geq k + l - (k_1 + l_1) \geq \frac{1}{2}(k + l). \tag{6}$$

Thus,

$$\begin{aligned} n &= k + l + s + t \\ &\leq 2(2st + (s + t)) + s + t \\ &= 4st + 3(s + t) \\ &\leq (s + t)^2 + 3(s + t). \end{aligned}$$

Solving for $s + t$, we obtain that

$$s + t \geq \frac{1}{2}\left(-3 + \sqrt{4n + 9}\right).$$

Thus, $\beta_D(G) = n - 2(s+t) \le n + 3 - \sqrt{4n+9}$.

If $\beta_D(G) = n + 3 - \sqrt{4n+9}$, then by the above analysis, we have $s = t$, $k = l = 2s(s+1)$ and $k_1 = l_1 = s(s+1)$. Moreover, each vertex of $X^+ \backslash X_1^+$ (respectively, $Y^+ \backslash Y_1^+$) has degree 2 and is joined to one vertex of Y^- and Y_1^+ (respectively, X^- and X_1^+), while each vertex of Y^- is joined to all vertices of X^- and $s+1$ vertices of $X^+ \backslash X_1^+$ and each vertex of X^- is joined to all vertices of Y^- and $s+1$ vertices of $Y^+ \backslash Y_1^+$. Thus, $\beta_D(G) = n + 3 - \sqrt{4n+9}$ implies that $G \in \mathcal{H}$.

If $G \in \mathcal{H}$, then $G = H_k$ for some $k \ge 2$. As G has order $n = 2k(2k+3)$, $k = \frac{1}{4}\left(\sqrt{4n+9} - 3\right)$. Assigning -1 to the $2k$ central vertices of the $(k+1)^*$-stars, and $+1$ to all other vertices, we define a BF f of G satisfying $f(V(G)) = 2k(2k+1) = n + 3 - \sqrt{4n+9}$. Hence, $\beta_D(G) \ge n + 3 - \sqrt{4n+9}$. It follows that $\beta_D(G) = n + 3 - \sqrt{4n+9}$. □

2.3 Regular graphs

We establish an upper bound of $\beta_D(G)$ for a regular graph in this subsection.

Theorem 4. *If G is k-regular graph of order n, then*

$$\beta_D(G) \le \begin{cases} 0 & \text{for } k \text{ even;} \\ n/k & \text{for } k \text{ odd.} \end{cases}$$

The upper bound in Theorem 4 is sharp, as will follow from Theorem 7.

Proof of Theorem 4. Let f be any BF of G. As G is a k-regular graph,

$$\sum_{v \in V(G)} f(N(v)) = kf(V(G)). \tag{7}$$

We discuss the following two cases.

*C*ase 1. k is odd.

As $f(N(v)) \le 1$ for each $v \in V(G)$, by (7), it follows that

$$kf(V(G)) \le n.$$

Hence, $\beta_D(G) \le n/k$.

*C*ase 2. k is even.

In this case, $|N(v)|$ is even for each $v \in V(G)$. So, for each $v \in V(G)$, $f(N(v)) \le 1$ implies that $f(N(v)) \le 0$. Thus,

$$\sum_{v \in V(G)} f(N(v)) \le 0.$$

By (7), it follows that $f(V(G)) \le 0$. Hence, $\beta_D(G) \le 0$. □

2.4 Classes of graphs

As the parameter $\beta'_S(G)$ is new, it is important to determine its values for some familiar graphs. For example, for cliques K_n when $n \geq 3$, we have that

$$\beta_D(K_n) = \begin{cases} 0 & \text{for } n \text{ even;} \\ -1 & \text{for } n \text{ odd.} \end{cases}$$

The exact values of $\beta_D(G)$ for cycles, paths and bicliques are found in this final subsection.

Theorem 5. *For any integer $n \geq 3$, we have*

$$\beta_D(C_n) = \begin{cases} 0 & \text{if } n \equiv 0 \pmod 4; \\ -2 & \text{if } n \equiv 2 \pmod 4; \\ -1 & \text{otherwise.} \end{cases}$$

Proof. As C_n is a 2-regular graph, by Theorem 4, we have $\beta_D(C_n) \leq 0$. Let $v_1, v_2, \ldots,$ v_n be n vertices of C_n in a clockwise order. We discuss the following four cases.

*C*ase 1. $n = 4k$ for some integer k. To show that $\beta_D(C_n) = 0$, it suffices to show that there is a BF f of C_n such that $f(V(C_n)) = 0$. In fact, assigning $+1, +1, -1, -1, \ldots,$ $+1, +1, -1, -1$ starting with v_1 clockwisely, we produce a BF f of C_n satisfying $f(V(C_n)) = 0$.

*C*ase 2. $n = 4k + 1$ for some integer k. To show that $\beta_D(C_n) \geq -1$, it suffices to show that there is a BF f of C_n such that $f(V(C_n)) = -1$. In fact, if we assign $1, 1, -1, -1, -1$ to the vertices in a clockwise order when $n = 5$, or assign $1, 1, -1, -1, \ldots, 1, 1, -1, -1, -1$ starting with v_1 clockwisely when $n \geq 9$, then we produce a BF f of C_n satisfying $f(V(C_n)) = -1$.

To show the other direction, we let f be a BF of C_n such that $f(V(C_n)) = \beta_D(C_n)$. Thus, $\beta_D(C_n) = |P| - |Q| = 2p - n \leq 0$. Therefore, $p \leq 2k$. It turns out that $\beta_D(C_n) = 2p - n \leq 2 * 2k - (4k + 1) = -1$.

*C*ase 3. $n = 4k+2$ for some integer k. It is straightforward to show that $\beta_D(C_n) = 0$ is impossible. Hence, $\beta_D(C_n) \leq -2$. To show that the equality holds, it suffices to show that there is a BF f of C_n such that $f(V(C_n)) = -2$. In fact, if we assign $1, 1, -1, -1, -1, -1$ to the vertices in a clockwise order when $n = 6$, or assign $1, 1, -1, -1 \ldots, 1, 1, -1, -1, -1, -1$ starting with v_1 clockwisely when $n > 6$, then we produce a BF f of C_n satisfying $f(V(C_n)) = -2$.

*C*ase 4. $n = 4k + 3$ for some integer k. Similar to Case 2, so we omit it. □

Theorem 6. *For any integer $n \geq 2$, we have*

$$\beta_D(P_n) = \begin{cases} 0 & \text{if } n \equiv 0 \pmod 4; \\ 2 & \text{if } n \equiv 2 \pmod 4; \\ 1 & \text{otherwise.} \end{cases}$$

Proof. Theorem 6 is obviously true for $2 \le n \le 4$. So we may assume that $n \ge 5$. Let $v_1, v_2, \ldots, v_n$ be n vertices of P_n with endvertices v_1 and v_n. We first show that $\beta_D(P_n) \le 2$. To do so, we take any BF f of P_n. For $i = 1$ or n, as $\deg(v_i) = 1$, we have that

$$f(N(v_i)) \le 1.$$

For $1 < i < n$, as $\deg(v_i) = 2$, we have that

$$f(N(v_i)) \le 0.$$

Thus,

$$\sum_{v \in V(G)} f(N(v)) \le 2.$$

Note that $\sum_{v \in V(G)} f(N(v)) = 2f(V(G)) - f(v_1) - f(v_n)$. So,

$$2f(V(G)) - f(v_1) - f(v_n) \le 2,$$

implying that $f(V(G)) \le 2$. Thus, $\beta_D(P_n) \le 2$.

Now we discuss the following four cases.

Case 1. $n = 4k$ for some positive integer k. The proof is straightforward, so is omitted.

Case 2. $n = 4k + 1$ for some positive integer k. In this case, $\beta_D(P_n) \le 2$ implies that $\beta_D(P_n) \le 1$. To show that the equality holds, it suffices to show there exists a BF f of P_n satisfying $f(V(G)) = 1$. In fact, we can produce such f by assigning $+1, +1, -1, -1, \ldots, +1, +1, -1, -1, 1$ to the vertices $v_1, v_2, \ldots, v_n$, respectively.

Case 3. $n = 4k + 2$ for some positive integer k. To show that $\beta_D(P_n) = 2$, it suffices to show there exists a BF f of P_n satisfying $f(V(G)) = 2$. In fact, we can produce such f by assigning $+1, +1, -1, -1, \ldots, +1, +1, -1, -1, 1, 1$ to the vertices $v_1, v_2, \ldots, v_n$, respectively.

Case 4. $n = 4k + 3$ for some positive integer k. Similar to Case 2, so we omit it. □

Theorem 7. *For any integer $n \ge 2$, we have*

$$\beta_D(K_{n,n}) = \begin{cases} 0 & \text{for } n \text{ even;} \\ 2 & \text{for } n \text{ odd.} \end{cases}$$

Proof. Let $X = \{u_1, u_2, \ldots, u_n\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$ be the partite sets of $K_{n,n}$. Observe that $K_{n,n}$ is an n-regular graph. We discuss the following two cases.

Case 1. n is even. By Theorem 4, $\beta_D(K_{n,n}) \le 0$. To show that $\beta_D(K_{n,n}) = 0$, it suffices to there exists a BF f of $K_{n,n}$ such that $f(V(K_{n,n})) = 0$. In fact, we can produce such f by assigning $f(v_i) = f(u_i) = (-1)^i, 1 \le i \le n$.

Case 2. n is odd. By Theorem 4, $\beta_D(K_{n,n}) \le 2$. To show that $\beta_D(K_{n,n}) = 2$, it suffices to there exists a BF f of $K_{n,n}$ such that $f(V(K_{n,n})) = 2$. In fact, we can construct such f by assigning $f(v_i) = f(u_i) = (-1)^{i+1}, 1 \le i \le n$. □

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