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4-restricted edge cuts of graphs*

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Abstract

A 4-restricted edge cut is an edge cut of a connected graph which disconnects the graph, where each component has order at least 4. Graphs that contain 4-restricted edge cuts are characterized in this paper. As a result, it is proved that a connected graph G of order at least 10 contains 4-restricted edge cuts if and only if it contains no cut-vertex u where every component of $G \setminus u$ has order at most 3.

1 Introduction

All graphs considered in this paper are simple and connected with order at least 8. When studying network reliability, one often considers a network model whose nodes never fail but whose edges fail independently with equal probability. Let M be such a kind of network, and denote by C_h the number of its edge cuts of size h. If M has size e and edge failure probability p, then its reliability is

$$R(M, p) = 1 - \sum_{h=1}^{e} C_h p^h (1-p)^{e-h}.$$

If one can determine all the coefficients C_h, then one can determine the reliability. But, unfortunately, Provan proved in [1] that it is NP-hard to determine all these coefficients. Employing super edge connectivity, Bauer [2] calculated the first λ coefficients C_h, where λ is the edge connectivity of M. To estimate more precisely the reliability, Esfahanian introduced the concepts of restricted edge cut and restricted edge connectivity in [3].

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Definition 1.1 A restricted edge cut is an edge cut of a connected graph which separates this graph into components without isolated vertices. Restricted edge connectivity is the size of a minimum restricted edge cut.

With the properties of restricted edge cut and restricted edge connectivity, Li determined the first $\lambda_2 - 1$ coefficients C_h of circulant graphs in [4], where λ_2 denotes the restricted edge connectivity. As was pointed out by Esfahanian, the restricted edge cut is also a useful tool for fault-tolerance analysis. But their results are fairly approximate. For more accurate results, we generalized these concepts in [5].

Definition 1.2 An m-restricted edge cut, or simply an R_m-edge cut, is an edge cut of a connected graph which separates this graph into components each having order at least m. The size of an m-restricted edge cut of graph G is called its m-restricted edge connectivity.

Denote by $\lambda_m(G)$ the m-restricted edge connectivity of a graph G. Clearly, a 2-restricted edge cut is the so-called restricted edge cut, and 2-restricted edge connectivity is restricted edge connectivity. For networks with the topology of regular graphs, Meng calculated the first $\lambda_3 - 1$ coefficients in [6] with the properties of 3-restricted edge cut and 3-restricted edge connectivity; Wang proved in [7] that networks with greater than 3-restricted edge connectivity and less than 3-restricted edge cut are more reliable under some reasonable conditions. Since R_m-edge cut and connectivity are important in their own right in the analysis of reliability and fault-tolerance, they draw a lot of attention; so we suggest that the reader refers to [8,9] for example. But so far no general criterion for the existence of an R_m-edge cut has been found apart from $m = 2, 3$ for some special graphs [10]. In this paper, we characterize those graphs that contain an R_4-edge cut by presenting the following:

Theorem 3.4 *Let G be a connected graph of order at least* 8*. Then G contains no R_4-edge cut if and only if $G \in F_{n,4} \cup \Omega_4^8 \cup \Omega_3^8 \cup \Omega_3^9$.*

These four graph collections are defined in Definitions 2.5, 3.1, 3.2 and 3.3 respectively. Before proceeding, we shall introduce some more symbols and terminology. Write $|S|$ for the cardinality of a set S or the order of a graph S. Let $\varepsilon = |E(G)|$ denote the size of a graph G and $c(G)$ its circumference. For two disjoint subsets A and B of $V(G)$, or two disjoint subgraphs of G, we denote by $[A, B]$ the set of edges with one endpoint in A and the other in B. Let $G \backslash A$ denote the graph obtained by removing all the vertices of A from G; we simplify $[A, G \setminus A]$ to $I(A)$. When A is an edge-set, $G - A$ means deleting the edges of A from G but leaving their endpoints. For a connected subgraph H of G, call every component of $G \backslash H$ a bridge of H, or simply an H-bridge. If B is an H-bridge, we refer to the vertices of H adjacent to B as the attachments of B. Write $A(B)$ for the set of attachments of a bridge B. For other symbols and terminology not explicitly stated, we follow [11].

2 Preliminaries

We begin with an interesting combinatorial phenomenon, which is the basis of the main result.

Lemma 2.1 *If we arrange $n + t$ boxes around a circle and put $2n$ balls into these boxes in such a way that no box is empty, then there are some consecutive boxes containing precisely n balls in total, where $n \geq t \geq 1$.*

Proof Denote by f an arbitrary arrangement of the $2n$ balls into these boxes. Since $n \geq t \geq 1$, we deduce from the pigeonhole principle that there are at least two boxes in f containing exactly one ball each. Label one of these two boxes with the number 0, and label the remaining boxes clockwise with $1, 2, \ldots, n+t-1$, respectively. Write $f(m)$ for the number of balls in the box labelled m (simply Box m). Define

$$F(m) = \sum_{i=0}^{m} f(i), \quad B(m) = \sum_{i=m}^{n+t-1} f(i) + f(0).$$

Let r and s be two integers in $\{0, 1, 2, ..., n + t - 1\}$ such that

$$F(s) = \max\{F(m) : n \geq F(m)\}, \quad B(r) = \max\{B(m) : n \geq B(m)\}.$$

Proposition A One of the following two statements is true.

1 $F(s) = n$ or $B(r) = n$.

2 $r = s + 2$, $n - 1 \geq F(s)$, $n - 1 \geq B(r)$.

Suppose Statement 1 is not true in some arrangement f. According to the definitions of $F(s)$ and $B(r)$, the two inequalities in Statement 2 are both true.

We claim first that $r \geq s + 2$. If this is not the case, then $F(s) + B(r) \geq 2n + 1$, which implies either $F(s) \geq n + 1$ or $B(r) \geq n + 1$, contradicting the choice of s and t.

We claim secondly that $r \leq s+2$. If not, $r \geq s+3$, and then $F(s+1)+B(r-1) \leq 2n + 1$, since $f(0)$ contributes twice to $F(s + 1) + B(r - 1)$ and $f(m)$ contributes at most once for every $m \neq 0$. This implies that

$$F(s + 1) \leq n \quad \text{or} \quad B(r - 1) \leq n. \tag{1}$$

But on the other hand, from the definition of $F(s)$ and $B(r)$, we have

$$F(s + 1) \geq n + 1 \quad \text{and} \quad B(r - 1) \geq n + 1. \tag{2}$$

The contradiction between (1) and (2) establishes the second claim. Proposition A follows from these two claims.

We continue to prove the lemma by induction on n. It is not difficult to check the truth of Lemma 2.1 when $n = 1$ or 2. Assume Lemma 2.1 holds for any number less than n. Clearly, it also holds when Statement 1 of Proposition A is true. Hence we need only consider the case when Statement 2 of Proposition A is true. Since $F(s+1) > n$ and $F(s) \leq n-1$, we have $f(s+1) \geq 2$. Take away from Box $s+1$ a ball, say b, and remove Box 0 together with its ball to get a new arrangement g of $n+t-1$ boxes and $2n-2$ balls. By the induction hypothesis, there is a set M of consecutive boxes that contains precisely $n-1$ balls in total.

We claim that at least one of Box 1 and Box $s+1$ is contained in M. Otherwise, we have $M \subset \{\text{Box } 2, \ldots, \text{Box } s\}$ or $M \subset \{\text{Box } s+2, \ldots, \text{Box } n+t-1\}$. It follows from Statement 2 of Proposition A that

$$\begin{aligned} g(M) &= \sum_{i \in M} g(i) \leq \max\{B(s+2)-1, F(s)-2\} \\ &\leq \max\{n-1-1, n-1-2\} = n-2. \end{aligned}$$

This is a contradiction. If M contains exactly one of Box 1 and Box $s+1$, we can get our desired consecutive boxes that contain exactly n balls in total in arrangement f by putting back ball b to Box $s+1$ or adding Box 0 together with its ball to M. If both Box 1 and Box $s+1$ are contained in M, then $\{\text{Box } 1, \ldots, \text{Box } s+1\} \subset M$ or $\{\text{Box } s+1, \text{Box } s+2, \ldots, \text{Box } n+t-1, \text{Box } 1\} \subset M$. Suppose $\{\text{Box } s+1, \text{Box } s+2, \ldots, \text{Box } 1\} \subset M$; then $g(M) \geq B(s+1) - 1 - f(0) + f(1) \geq n$, contradicting the fact that M contains precisely $n-1$ balls in total. This contradiction implies $\{\text{Box } 1, \ldots, \text{Box } s+1\} \subset M$. Suppose $\{\text{Box } 1, \ldots, \text{Box } s+2\} \subset M$; then $g(M) \geq F(s+2) - 1 - f(0) \geq n$, a contradiction. For the same reason, it is also impossible for $\{\text{Box } n+t-1, \text{Box } 1, \ldots, \text{Box } s+1\} \subset M$. Hence $M = \{\text{Box } 1, \ldots, \text{Box } s+1\}$. Now we can get the desired consecutive boxes that contain n balls in total from M by putting back ball b to Box $s+1$. Our proof is complete. □

Definition 2.2 Let r and s be two integers such that $r \geq 0$, $m+r \geq s \geq 1$, and define the graph collection

$$\begin{aligned} \Psi_{m+s}^{2m+r} &= \{G : G \text{ is a connected graph with } c(G) = m+s \\ &\qquad \text{and } \varepsilon(G) = |G| = 2m+r\}; \\ \Psi &= \bigcup_{r \geq 0, s \geq 1} \Psi_{m+s}^{2m+r}. \end{aligned}$$

Lemma 2.3 *If $G \in \Psi$, then G contains R_m-edge cuts.*

Proof The proposition is trivial when $m+r \geq s \geq m$, so assume, in the rest of the proof, that $m-1 \geq s \geq 1$. We are going to show that there are R_m-edge cuts in any graph G belonging to Ψ_{m+s}^{2m+r}, using induction on r. When $r=0$, let C be the unique cycle in G. If we regard every subgraph induced by the union of a C-bridge and its attachments as a box and the vertices it contains as balls, then, by Lemma 2.1, the preceding assertion is true. Suppose it is also true for any number less than r, $r \geq 1$. Since $m-1 \geq s$ and $r \geq 1$ in this case, there is at least one Box B that contains at least two balls by the pigeonhole principle. Hence the graph induced by the vertices in B is a tree T with order at least 2. Clearly, T contains at least one leaf u such that $G\backslash u \in \Psi_{m+s}^{2m+r-1}$. By the induction hypothesis, $G\backslash u$ contains an R_m-edge cut S; S is clearly an R_m-edge cut of G. □

Lemma 2.4 *Let G be a connected graph of order at least $2m$. If $c(G) \geq m+1$, then G contains R_m-edge cuts.*

Proof Let C be a longest cycle of G. By removing an edge-set S from G but conserving cycle C and the connecting property of G, we get a spanning subgraph H of G belonging to Ψ_{m+s}^{2m+r} for some r and s. The graph H contains an R_m-edge cut T by Lemma 2.3, which implies that the union of T and S is an R_m-edge cut of G. □

Definition 2.5 A flower F is a connected graph of order at least 8 which contains a cut-vertex w such that each component of $F\backslash w$ has order at most 3. We refer to vertex w as its stamen and the components of $F\backslash w$ as its petals.

Write $F_{n,4}$ for the set of flowers of order n. It is not difficult to see that every flower has only one stamen and at least three petals.

Lemma 2.6 *Let G be a tree of order at least 8. Then G contains R_4-edge cuts if and only if G is not a flower.*

Proof The necessity is obviously true. For the sufficiency, choose an edge $e = uv$ such that the order difference of the two components of $G-e$ is minimum. We claim that $\{e\}$ is an R_4-edge cut of G.

Suppose, to the contrary, that this is not the case. Assume without loss of generality that $u \in H$ and $v \in Q$, where H and Q are the two components of $G - e$ such that $|H| < |Q|$. Then $|H| < 4$. If there is a component D in $G \setminus v$ such that $|D| > |H|$, then $|D| < 4$. Otherwise, $||D| - |G\backslash D|| < |G\backslash H| - |H|$, which contradicts the choice of edge e. But now we see that G is a flower with stamen v. Our claim follows from this contradiction. □

3 Characterization

Definition 3.1 Let $P_5 = uvxyz$ be a path of length 4. Let w be an arbitrary vertex of another connected graph H of order 3. Then Ω_4^8 is the collection of graphs obtained from one of the following two different ways.

Method 1. Step 1 Join the graph H and path P_5 by adding two edges wv and wy to obtain a new graph N. This step results in three distinct graphs according to the choice of H and w.

Step 2 Add at most one of the two edges wx and vy to the graph N.

Method 2. Step 1 When H is a path with w as one of its pendants, join x to the degree 2 vertex of H after performing Step 1 of Method 1.

Step 2 Add at most one edge between w and x.

Definition 3.2 Graphs in the collection Ω_3^8 can be obtained as follows. Let A and B be two connected graphs of order 3, and let C be an isolated edge. Take three arbitrary vertices, one each from these three graphs, and join them into a 3-cycle.

Definition 3.3 The collection Ω_3^9 consists of graphs obtained by joining three vertices, one each from three arbitrary connected graphs of order 3, into a 3-cycle.

Remark It is not difficult to see that Ω_4^8 contains exactly eleven graphs of order 8 and circumference 4, Ω_3^8 contains precisely six graphs of order 8 and circumference 3, and Ω_3^9 contains in total ten graphs of order 9 and circumference 3.

Theorem 3.4 *A connected graph G of order at least 8 contains no R_4-edge cut if and only if $G \in F_{n,4} \cup \Omega_4^8 \cup \Omega_3^8 \cup \Omega_3^9$.*

Proof The sufficiency is obviously true. According to Lemma 2.6, the necessity is also true when G is a tree. Assume G is not a tree in the rest of this proof. From Lemma 2.4, it follows that $c(G) < 4 + 1 = 5$.

Case 1 Circumference $c(G) = 4$.
Let $C = uvxyu$ be a longest cycle of G. Suppose G is not a flower; we shall prove $G \in \Omega_4^8$ by the following eleven claims.

Claim 1 No C-bridge has order more than 3; G contains at least two C-bridges.

If G contains a C-bridge B of order at least 4, then $[B, C]$ is an R_4-edge cut. Claim 1 follows from this contradiction.

Claim 2 There are at least two C-bridges B_1 and B_2 in G such that $|A(B_1) \cup A(B_2)| \geq 2$.

Otherwise, according to the first part of Claim 1, G would be a flower with the unique attachment as its stamen, a contradiction.

Claim 3 The graph G contains no C-bridge of order more than 2.

If B is a counterexample, by Claim 2, the bridge B has an attachment u different from an attachment w of another C-bridge. Let z be a neighbour of u in B. Then $\{[B, C] - uz\} \cup \{uv, uy\}$ is an R_4-edge cut, a contradiction.

Claim 4 Let w be an arbitrary vertex in C. Then the C-bridges having attachment w have order sum at most 2.

Suppose, to the contrary, that the C-bridges having attachment $w = u$ have order sum at least 3. Call bridge B a k-bridge if it has exactly k attachments. If every C-bridge having attachment u is a 1-bridge, by Claim 2, $\{uv, uy\}$ is an R_4-edge cut. This contradiction shows that at least one of these C-bridges, say bridge B, is not a 1-bridge. Since $c(G) = 4$, bridge B is a 2-bridge. Assume without loss of generality that $A(B) = \{u, x\}$, and that $|B| = min\{|H| : H$ is a 2-bridge with $A(H) = \{u, x\}\}$.

If there exists a C-bridge having vertex v or y as its attachment, then the edge-set that separates G into two components, one of which is the subgraph induced by the union of u and all the C-bridges having attachment u, would be an R_4-edge cut of G. This contradiction shows that neither v nor y is an attachment of any C-bridge. Similarly, no C-bridge has x as its unique attachment.

Now we can construct an R_4-edge cut S as follows. If $|B| = 1$, let $S = [H, G\backslash H]$, where H is the subgraph of G induced by the union of $V(B)$ and $\{v, x, y\}$. If $|B| > 1$, by Claim 3, $|B| = 2$; let $S = [Q, G\backslash Q]$, where $Q = G[V(B) \cup \{x, y\}$. Claim 4 follows from these contradictions.

Claim 5 Any C-bridge is neither a 3-bridge nor a 4-bridge.

Otherwise we would obtain the contradiction that $c(G) > 4$.

Subcase 1.1 There exists a C-bridge B with $|A(B)| = 2$.

Claim 6 No other C-bridge is 2-bridge; $|B| = 2$. If $A(B) = \{u, x\}$, then neither u nor x is an attachment of any other C-bridges.

Suppose, to the contrary, that the C-bridge H is another 2-bridge. Since $c(G) = 4$, the two attachments of B (or H) are not adjacent to each other in C. Similarly, B and H have the same two attachments. Assume without loss of generality that $A(B) = A(H) = \{u, x\}$. From Claim 4, we deduce that $A(Q) \cap \{u, x\} = \emptyset$ for any C-bridge $Q \notin \{B, H\}$, and that $|B| = 1 = |H|$. If the vertex v is not an attachment of any C-bridge, let $P = G[V(B) \cup V(H) \cup \{u, v\}]$; then $S = [P, G\backslash P]$ is an R_4-edge cut. This contradiction shows that the vertex v is an attachment of a third C-bridge. Similarly, vertex y is also an attachment of some fourth C-bridge. Now let Q be the subgraph induced by the union of $\{v, u\}, V(B)$ and the vertex set of C-bridges having attachment v; $S = [Q, G\backslash Q]$ is again an R_4-edge cut. This contradiction shows that B is the unique 2-bridge.

If $|B| \neq 2$, by Claim 3, we have $|B| = 1$. Suppose there exists another C-bridge D such that $z \in \{u, x\} \cap A(D)$. By Claim 4, $|D| = 1$ and no other C-bridge has attachment u or x. Construct the R_4-edge cut S as follows. When $w \in \{v, y\}$ is not an attachment of some C-bridges, let $S = [P, G\backslash P]$, where $P = G[V(B) \cup V(D) \cup \{z, w\}]$. Otherwise, let $S = [Q, G\backslash Q]$, where Q is the subgraph of G induced by the union of $\{v, z\}, V(D)$ and the vertex set of C-bridges having attachment v. These contradictions show that $\{u, x\} \cap A(D) = \emptyset$ for any C-bridge $D \neq B$. Since $|G| \geq 8$, the preceding observation implies that there is a vertex $h \in \{v, y\}$ such that C-bridges having attachment h have order sum at least 2. By Claim 4, this order sum is 2. Let $S = [T, G\backslash T]$, where T is the subgraph of G induced by the union of $\{u, h\}$ and the vertex set of C-bridges having attachment h. Clearly, S is a 4-restricted edge cut. This contradiction shows that $|B| = 2$.

The third part follows directly from the combination of Claim 4 and the first two parts of Claim 6.

Claim 7 Let $N(u)$ denote the neighbourhood of vertex u in graph G. Then $|N(u) \cap V(B)| = 1$, $N(x) \cap V(B) = N(u) \cap V(B)$.

Otherwise, there is a path $P = uwzx$ in $G[\{u, x\} \cup V(B)]$. Clearly, $C_1 = uwzxvu$ is a cycle of length 5. This is impossible since $c(G) = 4$.

Claim 8 $G \in \Omega_4^8$.

By Claims 4, 5, 6 and 7, it suffices to show that $d(v) = d(y) = 3$ and $vy \notin E(G)$. The second part is obviously true since otherwise we would have $c(G) \geq 5$. In order to prove the first part, we show at first that $d(v) \geq 3$ and $d(y) \geq 3$. If this is not the case, then $d(v)$ or $d(y) = 2$, say $d(v) = 2$. Let $H = G[\{u, v\} \cup V(B)]$. Then $S = [H, G\backslash H]$ is an R_4-edge cut, a contradiction. We show secondly that $d(v) \leq 3$ and $d(y) \leq 3$. If $d(y) \geq 4$, the C-bridges having attachment y have order sum at least 2. Let T be the subgraph of G induced by the union of $\{u, v\}$, $V(B)$ and the vertex sets of C-bridges that have attachment v. Then $S = [T, G\backslash T]$ is an R_4-edge cut, also a contradiction. Claim 8 follows.

Subcase 1.2 No C-bridge is a 2-bridge.

Claim 9 There exists at least one vertex x in C such that no C-bridge has attachment x.

Otherwise, an R_4-edge cut can be easily found since every C-bridge is a 1-bridge in this case.

Claim 10 Cycle C has a unique vertex u such that the C-bridges having attachment u have order sum 2.

By Claims 4 and 9, there is at least one such vertex in C. If C contains at least two such vertices, the graph G contains an R_4-edge cut no matter whether these two vertices are adjacent to each other in C or not. This is a contradiction.

Claim 11 $G \in \Omega_4^8$.

Let u and x be the two vertices postulated in Claims 9 and 10. Then $ux \notin E(C)$, or otherwise, $S = [H, G\backslash H]$ is an R_4-edge cut, where H is the subgraph induced by the union of $\{u, x\}$ and the vertex sets of C-bridges that have attachment u, a contradiction. By Claim 10, $|G| = 8$. Combining this result with Claims 9 and 10, we conclude that each of the other two vertices in $C \setminus \{u, x\}$ is an attachment of exactly one C-bridge of order one. Let $C \setminus \{u, x\} = \{v, y\}$. Then $ux \notin E(G)$ or $vy \notin E(G)$, or otherwise another R_4-edge cut can be easily found. Claim 11 follows.

Case 2 Circumference $c(G) = 3$.
Assume that G is not a flower. We are going to show that $G \in \Omega_3^8$ or $G \in \Omega_3^9$. Let C be a 3-cycle of G. Then every C-bridge is a 1-bridge.

Claim 12 Let x be a vertex of C and B a C-bridge having attachment x. Then x has at most two neighbours in B.

If x has at least three neighbours in B, there is a path of length at least 2 joining some two neighbours of x in B. Therefore the graph G has a cycle of length at least

4. Claim 12 follows from this contradiction.

Consider at first the case when G contains only one C-bridge B. Let $A(B) = \{x\}$. By Claim 12, $|N(x) \cap V(B)| \leq 2$. Suppose $N(x) \cap V(B) = \{y\}$. Then $B \setminus y$ has a component F of order at least 4 since G is not a flower. It follows that $I(F)$ is a 4-restricted edge cut. This contradiction implies that $N(x) \cap V(B) = \{y, z\}$ for some two vertices y and z. If y is not a cut-vertex, then $N(y) = \{x, z\}$ since $c(G) = 3$. As a result, $I(B \setminus y)$ is an R_4-edge cut. This contradiction shows that both y and z are cut-vertices. If the union F of the components of $G \setminus y$ not containing C has order at least 3, then $I(G[F \cup \{y\}])$ is a 4-restricted edge cut, a contradiction. Similarly, the union of components of $G \setminus z$ not containing C has order at most 2. Hence $8 \leq |G| \leq |C| + |\{y, z\}| + 2 + 2 = 9$. From the above discussion, we conclude that $G \in \Omega_3^8$ or Ω_3^9.

Consider secondly that the graph G contains at least two C-bridges. In this case, we have

Claim 13 Every C-bridge has order at most 3.

If all the C-bridges have a common attachment x, then $G \setminus x$ contains no components of order more than 3 by Claim 13. It follows that G is a flower with stamen x. This contradiction shows that

Claim 14 There exist two C-bridges B and H such that $A(B) \neq A(H)$.

Claim 15 Let z be a vertex of C. Then the order sum of C-bridges that have attachment z is not more than 2. Hence $8 \leq |G| \leq 9$.

Let F be the subgraph induced by the union of vertex z and the C-bridges having attachment z. Since G is not a flower, the component of $G \setminus z$ that contains $C \setminus z$ has order at least 4 by Claim 13. It follows that if the first part of Claim 15 is not true, then $[F, G \setminus F]$ is an R_4-edge cut, which is a contradiction. The second part of Claim 15 follows directly from the first one.

If $|G| = 8$, by Claim 14 and the first part of Claim 15, the cycle C contains a unique vertex z such that the order sum of C-bridges having attachment z is equal to one. Therefore $G \in \Omega_3^8$.

If $|G| = 9$, then, for any vertex z of C, the order sum of C-bridges that have attachment z is 2 by Claim 14 and the first part of Claim 15. Therefore $G \in \Omega_3^9$. Theorem 3.4 follows. □

The following corollary is a direct result of Theorem 3.4.

Corollary 3.5 *Let G be a connected graph of order at least* 10. *Then G contains an R_4-edge cut if and only if G is not a flower.*

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