AUSTRALASIAN JOURNAL OF COMBINATORICS
Volume **30** (2004), Pages 41–49

2-restricted edge connectivity of vertex-transitive graphs*

Jun-Ming Xu   Qi Liu

Department of Mathematics
University of Science and Technology of China
Hefei, Anhui, 230026
P. R. CHINA
xujm@ustc.edu.cn

Abstract

The 2-restricted edge-connectivity λ'' of a graph G is defined to be the minimum cardinality $|S|$ of a set S of edges such that $G-S$ is disconnected and is of minimum degree at least two. It is known that $\lambda'' \le g(k-2)$ for any connected k-regular graph G of girth g other than K_4, K_5 and $K_{3,3}$, where $k \ge 3$. In this paper, we prove the following result: For a connected vertex-transitive graph of order $n \ge 7$, degree $k \ge 6$ and girth $g \ge 5$, we have $\lambda'' = g(k-2)$. Moreover, if $k \ge 6$ and $\lambda'' < g(k-2)$, then $\lambda''|n$ or $\lambda''|2n$.

1 Introduction

In this paper, a graph $G = (V, E)$ always means a simple undirected graph (without loops and multiple edges) with vertex-set V and edge-set E. We follow Bondy and Murty [1] or Xu [18] for graph-theoretical terminology and notation not defined here.

It is well-known that when the underlying topology of an interconnection network is modelled by a graph G, the connectivity of G is an important measure for fault-tolerance of the network [17]. However, this measure has many deficiencies (see [2]). Motivated by the shortcomings of the traditional connectivity, Harary [5] introduced the concept of conditional connectivity by requiring some specific conditions to be satisfied by every connected component of $G-S$, where S is a minimum cut of G. Certain properties of connected components are particularly important for applications in which parallel algorithms can run on subnetworks with a given topological structure [2, 6]. In [2, 3], Esfahanian and Hakim proposed the concept of restricted connectivity by requiring that very connected component must contain no isolated

* The work was supported by ANSF (No. 01046102) and NNSF of China (No. 10271114).

vertex. The restricted connectivity can provide a more accurate fault-tolerance measure of networks and have received much attention recently. (For example, see [2, 3], [6]-[10], [14]-[19].) For regular graphs Latifi *et al* [6] generalized the restricted connectivity to h-restricted connectivity for the case of vertices by requiring that every connected component contains no vertex of degree less than h. In this paper we are interested in similar kind of connectivity for the case of edges.

Let h be a nonnegative integer. Let G be a connected graph with minimum degree $k \geq h+1$. A set S of edges of G is called an *h-restricted edge-cut* if $G-S$ is disconnected and is of minimum degree at least h. If such an edge-cut exists, then the *h-restricted edge-connectivity* of G, denoted by $\lambda^{(h)}(G)$, is defined to be the minimum cardinality over all h-restricted edge-cuts of G. From this definition, it is clear that if $\lambda^{(h)}$ exists, then for any l with $0 \leq l \leq h$, $\lambda^{(l)}$ exists and

$$\lambda^{(0)} \leq \lambda^{(1)} \leq \cdots \leq \lambda^{(l)} \leq \cdots \leq \lambda^{(h)}.$$

It is clear that $\lambda^{(0)}$ is the traditional edge-connectivity and $\lambda^{(1)}$ is the restricted edge-connectivity defined in [2, 3]. In this paper, we restrict ourselves to $h=2$. For the sake of convenience, we write λ'' for $\lambda^{(2)}$. We use $g=g(G)$ to denote the girth of G, that is, the length of a shortest cycle in G. The following result ensures the existence of $\lambda''(G)$ if G is regular.

Theorem 1 (Xu [15]) *Let G be a connected k-regular graph with girth g other than K_4, K_5 and $K_{3,3}$, where $k \geq 3$. Then $\lambda''(G)$ exists and $\lambda''(G) \leq g(k-2)$.*

A graph G is called *vertex-transitive* if there is an element π of the automorphism group $\Gamma(G)$ of G such that $\pi(x) = y$ for any two vertices x and y of G. It is well-known [12, 13] that the edge-connectivity of a vertex-transitive graph is equal to its degree. The restricted edge-connectivity of vertex-transitive graphs has been studied in [16, 19].

For a special class of vertex-transitive graphs, circulant graphs, its 2-restricted edge-connectivity has been determined by Li [9]. In [14] Xu proved that $\lambda''(G) = g(k-2)$ for a vertex-transitive graph $G\,(\neq K_5)$ with even degree k and girth $g \geq 5$. In this paper, we prove the following result by making good use of the technique proposed by Mader [12] and Watkins [13], independently.

Theorem 2 *For a connected vertex-transitive graph of order $n \geq 7$, girth g and degree $k\,(\geq 4$ and $\neq 5)$, if $g \geq 5$ we have $\lambda'' = g(k-2)$. Moreover, if $\lambda'' < g(k-2)$, then $\lambda'' | n$ or $\lambda'' | 2n$.*

Note that in this theorem k is not required to be even. The proof of Theorem 2 will be given in Section 3, and this follows the proof of two lemmas in the next section.

2 Notation and Lemmas

Let G be a k-regular graph, where $k \geq 2$. Then G contains a cycle and hence its girth is finite. It is known (see [11, Problem 10.11]) that

$$|V(G)| \geq f(k,g) = \begin{cases} 1 + k + k(k-1) + \cdots + k(k-1)^{(g-3)/2}, & \text{if } g \text{ is odd;} \\ 2[1 + (k-1) + \cdots + (k-1)^{(g-2)/2}], & \text{if } g \text{ is even.} \end{cases} \quad (1)$$

A vertex x of G is called *singular* if it is of degree zero or one. Let X and Y be two distinct nonempty proper subsets of V. The symbol (X, Y) denotes the set of edges between X and Y in G. If $Y = \overline{X} = V \setminus X$, then we write $\partial(X)$ for $(X, \overline{X})$ and $d(X)$ for $|\partial(X)|$. The following inequality is well-known (see [11, Problem 6.48]).

$$d(X \cap Y) + d(X \cup Y) \leq d(X) + d(Y). \quad (2)$$

A 2-*restricted edge-cut* S of G is called a λ''-*cut* if $|S| = \lambda''(G) > 0$. Let X be a proper subset of V. If $\partial(X)$ is a λ''-cut of G, then X is called a λ''-*fragment* of G. It is clear that if X is a λ''-fragment of G, then so is $\overline{X}$ and both $G[X]$ and $G[\overline{X}]$ are connected. A λ''-fragment X is called a λ''-*atom* of G if it has the minimum cardinality. It is clear that G certainly contains λ''-atoms if $\lambda''(G)$ exists. For a given λ''-atom X of G, since $G[X]$ is connected and contains no singular vertices, it contains a cycle. Thus $g(G) \leq |X| \leq |V(G)|/2$.

Lemma 3 *Let G be a connected k-regular graph, where $k \geq 3$. Let R be a proper subset of $V(G)$ and U be the set of singular vertices in $G - \partial(R)$. If $\lambda''(G)$ exists and $U \subseteq R$, then $|R| < g(G)$ provided that one of the following three conditions is satisfied:*

(a) $d(R) \leq \lambda''(G)$;

(b) $d(R) \leq \lambda''(G) + 1$ and $|U| \geq 2$ or $k \geq 4$;

(c) $d(R) \leq \lambda''(G) + 1$ and $|U| = 1, k = 3$, and R contains no λ''-fragments of G.

Proof Let $g = g(G)$. Since $\lambda''(G)$ exists, $\lambda''(G) \leq g(k-2)$ by Theorem 1. Suppose to the contrary that $|R| \geq g$. We will derive contradictions.

If $G[R]$ contains no cycles, then $|E(G[R])| \leq |R| - 1$ and

$$\begin{aligned} g(k-2) + 1 &\geq \lambda''(G) + 1 \geq d(R) = |R|k - 2|E(G[R])| \\ &\geq |R|k - 2(|R| - 1) = |R|(k-2) + 2 \\ &\geq g(k-2) + 2, \end{aligned}$$

which is a contradiction.

In the following we assume that $G[R]$ contains cycles. Let R' be the vertex-set of the union of all maximal 2-connected subgraphs of $G[R]$. Then $U \subseteq R \setminus R'$. Note that for any two distinct vertices u and v in R', any neighbor of u and any neighbor of v in $R \setminus R'$ are not joined by a path. This implies that $G - \partial(R')$ contains no singular vertices. So $\partial(R')$ is a λ''-restricted edge-cut of G for which $d(R') \geq \lambda''(G)$. Also note that for any edge $e \in (R', R \setminus R')$, either e is incident with some vertex $z \in U$ or there is a path in $G[R \setminus R']$ connecting e to some vertex $z \in U$. Furthermore, if two edges $e, e' \in (R', R \setminus R')$ are distinct, then the corresponding two vertices $z, z' \in U$

are distinct too. Thus $|(R', R \setminus R')| \le |U|$, and $|(R \setminus R', \overline{R})| \ge |U|(k-1)$ since $U \subseteq R \setminus R'$. It follows that

$$\begin{aligned} d(R') &= d(R) - |(R \setminus R', \overline{R})| + |(R', R \setminus R')| \\ &\le d(R) - |U|(k-1) + |U| \\ &= d(R) - |U|(k-2), \end{aligned}$$

from which we have

$$\lambda''(G) \le d(R') \le d(R) - |U|(k-2). \tag{3}$$

If $d(R) \le \lambda''(G)$, then from (3) we have $\lambda''(G) \le d(R) - 1 \le \lambda''(G) - 1$, which is a contradiction.

If $d(R) \le \lambda''(G) + 1$ and $|U| \ge 2$ or $k \ge 4$, then from (3) we have $\lambda''(G) \le d(R) - 2 = \lambda''(G) - 1$, again a contradiction.

If $d(R) \le \lambda''(G) + 1, |U| = 1, k = 3$, then from (3), we have $d(R') = \lambda''(G)$. Thus R' is a λ''-fragment of G contained in R, which contradicts our condition (c). The proof of the lemma is complete. ∎

Lemma 4 *Let G be a connected k-regular graph with $\lambda''(G) < g(k-2)$, where $k \ge 3$. If X and X' are two distinct λ''-atoms of G, then $|X \cap X'| < g$. Moreover, $X \cap X' = \emptyset$ for any k with $k \ge 4$ and $k \ne 5$.*

Proof Note that $|X| \ge g$ since X is a λ''-atom of G. If $|X| = g$, then $G[X]$ is a cycle of length g. Thus $g(k-2) = d(X) = \lambda''(G) < g(k-2)$, a contradiction. So we have $|X| > g$. Let

$$A = X \cap X', \quad B = X \cap \overline{X'}, \quad C = \overline{X} \cap X' \quad D = \overline{X} \cap \overline{X'}.$$

Then $|D| \ge |A|$ and $|B| = |C| = |X| - |A| \ge 1$ since X and X' are two distinct λ''-atoms of G.

We first show $|A| < g$. In fact, if $d(A) \le \lambda''(G)$, then $G - \partial(A)$ contains singular vertices (for otherwise, A is a λ''-fragment whose cardinality is smaller than $|X|$), and all of them are contained in A. Thus, $|A| < g$ by Lemma 3. If $d(A) > \lambda''(G)$, then

$$d(D) = d(X \cup X') \le d(X) + d(X') - d(X \cap X') < \lambda''(G),$$

which implies that $G - \partial(D)$ contains singular vertices (for otherwise, D is a 2-restricted edge-cut whose cardinality is smaller than λ''), and all of them are contained in D. Thus, $|D| < g$ by Lemma 3, and so $|A| \le |D| < g$.

We now show $|A| = 0$ for any k with $k \ge 4$ and $k \ne 5$. Suppose to the contrary that $|A| > 0$. Since $|A| < g$, $G[A]$ contains no cycle, that is, $G - \partial(A)$ contains at least one singular vertex. Let y be a singular vertex in $G - \partial(A)$. Then $y \in A$. Consider the set $X \setminus \{y\}$ if $|(y, C)| > |(y, B)|$, and the set $X' \setminus \{y\}$ if $|(y, C)| < |(y, B)|$. Then

$$d(X \setminus \{y\}) \le d(X) - |(y, D)| - |(y, C)| + |(y, B)| + 1 \le d(X) = \lambda''(G). \tag{4}$$

So there are singular vertices in $G - \partial(X \setminus \{y\})$, and all of them are in $X \setminus \{y\}$. By Lemma 3, $|X \setminus \{y\}| < g$, and so $g < |X| = |X \setminus \{y\}| + 1 \le g$, a contradiction. Thus, we need only to consider the case where $|(y, C)| = |(y, B)|$. Note that in this case the inequality (4) does not hold only when $|(y, D)| = 0$ and y is a vertex of degree one in $G - \partial(A)$. It follows that $k = d_G(y) = |(y, C)| + |(y, B)| + 1$. Thus, we need only to consider the case where k is odd.

Let W be the vertex-set of the connected component of $G[A]$ that contains y. Note that W contains at least two vertices of degree one in $G - \partial(A)$, and that $W \subseteq A$. Thus, $2 \leq |W| < g$. Let $Y = X \setminus W$ if $|(W, B)| \leq |(W, C)|$, and $Y = X' \setminus W$ if $|(W, B)| \geq |(W, C)|$. Then $\emptyset \neq Y \subset X$. Then

$$d(Y) = d(X) - |(W, C)| - |(W, D)| + |(W, B)| \leq d(X) = \lambda''(G)$$

which implies $|Y| < g$ by Lemma 3.

Since k is odd and is at least 7, there are at least 3 neighbors of y in B and C, respectively. We claim that no two neighbors of y are in the same component of $G[Y]$. Suppose to the contrary that some component of $G[Y]$ contains at least two neighbors of y. Choose two such vertices y_1 and y_2 so that their distance in $G[Y]$ is as short as possible. Let P be a shortest y_1y_2-path in $G[Y]$. Clearly, P does not contain any other neighbors of y except y_1 and y_2. Thus the length of P satisfies $\varepsilon(P) \leq |Y| - 2 \leq g - 3$, and so the length of the cycle $yy_1 + P + y_2y$ is smaller than g, a contradiction.

Thus, all neighbors of y in Y are in different components of $G[Y]$. Since $|Y| < g$, we can choose such a component H of $G[Y]$ so that its order is at most $\lfloor \frac{1}{3}\, g \rfloor$. Let $z \in V(H)$ be a neighbor of y. Then z is in B. Moreover, we claim that $d_H(z) \geq 2$. In fact, if z is a singular vertex in $G[H]$, then $d(X') \leq \lambda''(G) + 1$ and all singular vertices of $G - \partial(X')$ are in X', where $X' = X \setminus \{y\}$. By Lemma 3, $|X| - 1 < g$, that is, $|X| \leq g$, a contradiction.

Let L be a longest path containing z in H with two distinct end-vertices a and b. Then the length of L is at most $\lfloor \frac{1}{3}\, g \rfloor - 1$. Noting that $d_H(a) = d_H(b) = 1$, it follows that there exist $c, d \in W \setminus \{y\}$ such that they are neighbors of a and b, respectively. If $c = d$, then the length of the cycle $ac + cb + L$ is equal to $2 + \varepsilon(L) \leq 2 + \lfloor \frac{1}{3}\, g \rfloor - 1 < g$, which is impossible. Therefore, we have $c \neq d$.

Let Q and R be the unique yc-path and yd-path in $G[W]$ since $G[W]$ is a tree, and let e be the last common vertex of Q and R starting with y. Note that $e \neq y$ and

$$\varepsilon(Q) + \varepsilon(R) + \varepsilon(Q(c, e) \cup R(e, d)) = 2[\varepsilon(Q) + \varepsilon(R(e, d))] \leq 2(g - 2).$$

Therefore, at least one of $\varepsilon(Q)$, $\varepsilon(R)$ and $\varepsilon(Q(c, e) \cup R(e, d))$ is at most $\lfloor \frac{2}{3}\,(g - 2) \rfloor$.

If $\varepsilon(Q) \leq \lfloor \frac{2}{3}\,(g-2) \rfloor$, then, by considering the lengths of the cycle $C_1 = L(a, z) + yz + Q + ca$, we have

$$g \leq \varepsilon(C_1) \leq \left(\left\lfloor \frac{g}{3} \right\rfloor - 2 \right) + 2 + \left\lfloor \frac{2(g-2)}{3} \right\rfloor \leq g - 1,$$

a contradiction.

If $\varepsilon(R) \le \lfloor \frac{2}{3}(g-2) \rfloor$, then, by considering the lengths of the cycle $C_2 = zy + R + db + L(d,z)$, we have

$$g \le \varepsilon(C_2) \le 2 + \left\lfloor \frac{2(g-2)}{3} \right\rfloor + \left(\left\lfloor \frac{g}{3} \right\rfloor - 2 \right) \le g-1,$$

again a contradiction.

If each of $\varepsilon(Q)$ and $\varepsilon(R)$ is more than $\lfloor \frac{2}{3}(g-2) \rfloor$, then $\varepsilon(Q(c,e) \cup R(e,d)) \le \lfloor \frac{2}{3}(g-2) \rfloor$ and by considering the lengths of the cycle $C_3 = ac + Q(c,e) \cup R(e,d) + db + L$, we have

$$g \le \varepsilon(C_3) \le 2 + \left\lfloor \frac{2(g-2)}{3} \right\rfloor + \left(\left\lfloor \frac{g}{3} \right\rfloor - 1 \right) \le g-1,$$

a contradiction.

The proof of Lemma 4 is complete. ∎

3 Proof of Theorem 2

Let G be a connected vertex-transitive graph with order n (≥ 7) and degree k (≥ 4 and $\ne 5$). Then $\lambda''(G)$ exists and $\lambda''(G) \le g(k-2)$ by Theorem 1. Suppose that $\lambda''(G) < g(k-2)$, and let X be a λ''-atom of G. Under these assumptions we prove the following claims.

Claim 1 $G[X]$ is vertex-transitive.

Proof Let x and y be any two vertices in X. Since G is vertex-transitive, there is $\pi \in \Gamma(G)$ such that $\pi(x) = y$. Denote $\pi(X) = \{\pi(x) : x \in X\}$. It is clear that $G[X] \cong G[\pi(X)]$ because π induces an isomorphism between $G[X]$ and $G[\pi(X)]$. Hence $\pi(X)$ is also a λ''-atom of G. Since $y \in X \cap \pi(X)$, by Lemma 4, $X = \pi(X)$. Thus, the setwise stabilizer

$$\Pi = \{\pi \in \Gamma(G) : \pi(X) = X\}$$

is a subgroup of $\Gamma(G)$, and the constituent of Π on X acts transitively. This shows that $G[X]$ is vertex-transitive. ∎

Claim 2 There exists a partition $\{X_1, X_2, \cdots, X_m\}$ of $V(G)$, where $m \ge 2$, such that $G[X_i] \cong G[X]$ and X_i is a λ''-atom for $i = 1, 2, \cdots, m$.

Proof Let x be a fixed vertex in X. Let u be any element in $\overline{X}$. Since G is vertex-transitive, there exists $\sigma \in \Gamma(G)$ such that $\sigma(x) = u$. Moreover, $\sigma(X)$ is a λ''-atom of G. Let $X_u = \sigma(X)$. Then $X \cap X_u = \emptyset$ by Lemma 4 and $G[X] \cong G[X_u]$. Thus there are at least two λ''-atoms of G. It follows that for every u in G there is a λ''-atom X_u that contains u such that $G[X_u] \cong G[X]$, and either $X_u = X_v$ or $X_u \cap X_v = \emptyset$ for any two distinct vertices u and v of G. These λ''-atoms, $X_1, X_2, \cdots, X_m$, form a partition of $V(G)$, and $G[X_i] \cong G[X]$, $i = 1, 2, \cdots, m$. Since G has at least two distinct λ''-atoms, we have $m \ge 2$. ∎

Claim 3 $g = 3$ or 4 and $\lambda'' | n$ or $\lambda'' | 2n$.

Proof Suppose that $\lambda''(G) < g(k-2)$ and X is a λ''-atom of G. Then $G[X]$ is vertex-transitive by Claim 1 and there exists a divisor $m\,(\geq 2)$ of n such that $|X| = n/m$ by Claim 2. Let t denote the degree of $G[X]$. Then $2 \leq t \leq k-1$ and

$$\lambda''(G) = d(X) = |\partial(X)| = (k-t)|X| = (k-t)n/m. \tag{5}$$

Since $G[X]$ contains a cycle of length at least g, it follows from (1) and (5) that

$$g(k-2) > \lambda''(G) = (k-t)|X| \geq (k-t)f(t,g). \tag{6}$$

Case 1 g is even. In this case, from (1) and (6), we have

$$0 < g(k-2) - (k-t)2[1 + (t-1) + \cdots + (t-1)^{(g-2)/2}]. \tag{7}$$

The right hand side of (7) is increasing with respect to t and is decreasing with respect to g. It is not difficult to show that the inequality (7) can hold only when $g = 4$ and $t = k-1$. So $\lambda''(G) = |X| = n/m$ by (5).

Case 2 g is odd. In this case, from (1) and (6), we have

$$0 < g(k-2) - (k-t)[1 + t + t(t-1) + \cdots + t(t-1)^{(g-3)/2}]. \tag{8}$$

The right hand side of (8) is increasing with respect to t and is decreasing with respect to g. It is not difficult to show that the inequality in (8) can hold only when $g = 3$ and $t = k-2$ or $t = k-1$. If $t = k-1$, then $\lambda''(G) = |X| = n/m$ by (5). If $t = k-2$, then $\lambda''(G) = 2|X| = 2n/m$ by (5). ∎

From Claim 3, it follows that, if $g \geq 5$, then $\lambda'' = g(k-2)$. Also, if $\lambda''(G) < g(k-2)$, then $g = 3$ or 4, and hence $\lambda''|n$ or $\lambda''|2n$. The proof of Theorem 2 is complete.

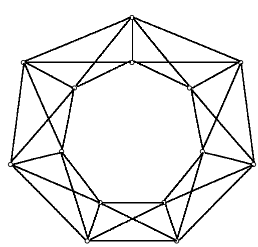

Figure 1: A vertex-transitive graph of degree $k = 5$ and $\lambda'' = 8$

Remarks The result $\lambda''(G) = g(k-2)$ is invalid for connected vertex-transitive graphs of degree $k = 5$. For example, consider the lexicographical product $C_n[K_2]$ of C_n by K_2, where C_n is a cycle of order $n \geq 4$, K_2 is a complete graph of order two. The definition of lexicographical product of graphs is referred to [4, pp.21-22] and the graph shown in Figure 1 is $C_7[K_2]$. Since both C_n and K_2 are vertex-transitive, $C_n[K_2]$ is vertex-transitive (see, [4, the exercise 14.19]). It is easy to see that $C_n[K_2]$

is of degree $k = 5$, girth $g = 3$ and a set of any four vertices that induce a complete graph K_4 is a λ''-atom of $C_n[K_2]$, and hence $\lambda'' = 8 < 3(5-2)$. Two distinct λ''-atoms X and X' corresponding two complete graphs of order four with an edge in common satisfy $|X \cap X'| = 2 < 3 = g$. This fact shows that the latter half of Lemma 4 is invalid for $k = 5$.

Acknowledgement

The authors would like to thank the anonymous referees for their valuable suggestions in order to improve the final version of the paper.

References

[1] J.A. Bondy. and U.S.R. Murty, *Graph Theory with Applications*, Macmillan Press, London, 1976.

[2] A.H. Esfahanian, Generalized measures of fault tolerance with application to N-cube networks, *IEEE Trans. Comput.* **38** (11) (1989), 1586–1591.

[3] A.H. Esfahanian and S.L. Hakimi, On computer a conditional edge- connectivity of a graph, *Information Processing Letters* **27** (1988), 195–199.

[4] F. Harary, *Graph Theory*, Addison-Wesley Publishing Company, Inc. 1969.

[5] F. Harary, Conditional connectivity, *Networks* **13** (1983), 346–357.

[6] S. Latifi, M. Hegde and M. Naraghi-Pour, Conditional connectivity measures for large multiprocessor systems, *IEEE Trans. Comput.* **43** (1994), 218–221.

[7] Q.L. Li and Q. Li, Reliability analysis of circulant graphs, *Networks*, **31** (1998), 61–65.

[8] Q.L. Li and Q. Li, Refined connectivity properties of abelian Cayley graphs, *Chin. Ann. of Math.* **19B** (1998), 409–414.

[9] Q.L. Li, Graph theoretical studies on fault-tolerance and reliability of networks (Chinese), Ph.D. Thesis, University of Science and Technology of China, 1997.

[10] Q. Li and Y. Zhang, Restricted connectivity and restricted fault diameter of some interconnection networks, *DIMACS* **21** (1995), 267–273.

[11] L. Lovász, *Combinatorial Problems and Exercises*, North Holland Publishing Company, Amsterdam/New York/Oxford, 1979.

[12] W. Mader, Eine Eigenschft der Atome andlicher Graphen. *Archives of Mathematics* (Basel) **22** (1971), 333–336

[13] A.E. Watkins, Connectivity of transitive graphs, *J. Combin. Theory* **8** (1970), 23–29.

[14] J.-M. Xu, Some results of R-edge-connectivity of even regular graphs, *Applied Math. J. Chinese Univ.* **14B** (3) (1999), 366–370.

[15] J.-M. Xu, On conditional edge-connectivity of graphs. *Acta Math. Appl. Sinica*, B **16** (4) (2000), 414–419.

[16] J.-M. Xu, Restricted edge-connectivity of vertex-transitive graphs (Chinese), *Chinese Annals of Math.*, **21A** (5) (2000), 605-608; An English version in *Chinese J. Contemporary Math.*, **21** (4) (2000), 369–374.

[17] J.-M. Xu, *Toplogical Structure and Analysis of Interconnection Networks*, Kluwer Academic Publishers, Dordrecht/Boston/London, 2001.

[18] J.-M. Xu, *Theory and Application of Graphs*, Kluwer Academic Publishers, Dordrecht/Boston/London, 2003

[19] J.-M. Xu, and K.-L. Xu, On restricted edge-connectivity of graphs, *Discrete Math.* **243** (2002), 291–298.

(Received 29 Sep 2002; revised 20 Jan 2004)