The ambiguity problem arising in multisensor data association

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Abstract

In many applications, (for example: air traffic control; robotics; ocean surveillance and medical diagnostics) one needs to process and analyse large volumes of data. The multisensor data association problem is concerned with determining which measurements from one or more sensors actually relate to the same object. This problem can be formulated as a multidimensional assignment problem. In the application of bearing-only data association, the solution of this NP-hard problem gives a list of targets and their position. Unfortunately, this solution though mathematically correct may not be physically valid as false targets known as ghosts may occur. This situation occurs whenever the number of targets is greater than or equal to the number of sensors. When ghosts are present, the ambiguity problem can arise. The ambiguity problem is the focus of this paper and we establish bounds on various sensor-target configurations.

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1 Introduction

Data integration is a process that advanced species carry out on a day-to-day basis. For example, the human brain processes information from the five sensors, eyes, skin, ears, tongue and nose, to sample the environment, build an awareness of it, and respond to observed changes in it. The process involved with using observations from a number of sensors to build a coherent awareness and ability to respond is referred to as multisensor data fusion.

Increasing technological advances in areas such as sensors, signal processing, high performance computing and communications have made emulating the process of multisensor data fusion practical ([15]). The military featured prominently in early applications of data fusion such as: battlefield surveillance; automated target identification and target tracking. Recently, the methods have been applied to non-military situations such as monitoring of manufacturing processes; medical diagnosis; robotics; and smart buildings ([8] and [16]).

A fundamental problem that arises in data fusion is that of utilising the data for detection and localisation of objects. The underlying problem is referred to as the data association problem. The characteristics of the application will determine how the data association is carried out. For example, if the sensors are commensurate (eg. two sonars, a towed array and a flank array) then the measurements can be directly combined. That is, on a measurement-to-measurement basis. However, for non-commensurate sensors (eg. a radar and a sonar), the data can be combined at a measurement-to-track or track-to-track basis. Note that a track is a sequence of measurements (of the same entity) over a specified time period.

The data association problem can be formulated in many different ways ([6], [9], [11], [12], [13] and [17]). One formulation is as a multi-dimensional generalized assignment problem. For $m \geq 3$ sensors, the problem is computationally difficult (NP hard). In the case of $m = 3$ sensors, the data association problem is formulated as a three-dimensional assignment problem in [11] and [12], where the cost coefficients are obtained by maximising the non-linear joint likelihood function of the measurement partition.

The three-dimensional assignment problem has been the focus of considerable attention as it occurs in many application areas, not just data association. A number of algorithms, both exact and heuristic have been proposed. Exact methods include Lagrangian relaxation (see [7], [11] and [14]) and Branch and Bound (see [4]). Tabu search (see [10]), greedy, max-regret and reduced cost (see [2] and [4]) are some of the heuristics that have been applied. Computational results, based on 300 randomly generated problems reported in [1], [2] and [3] demonstrate that good solutions can be obtained for practical sized problems.

The output from solving the data association problem using either heuristics or exact methods is a list of targets and an estimate of their position. This list is referred to as a hard assignment because once fixed, they cannot be changed. The other important thing to realise is that finding the list is not the end and that further problems may still exist. One such problem is that of ambiguity in which the solution presented may be mathematically correct but is physically invalid. This is referred to as the ambiguity problem and is illustrated in Figure 1.1 where there are two sensors (S_1 and S_2) and two targets. Here, without prior information you cannot declare which points, $\{A, D\}$ or $\{B, C\}$ actually relate to the true targets. Note that only one target can exist per line.

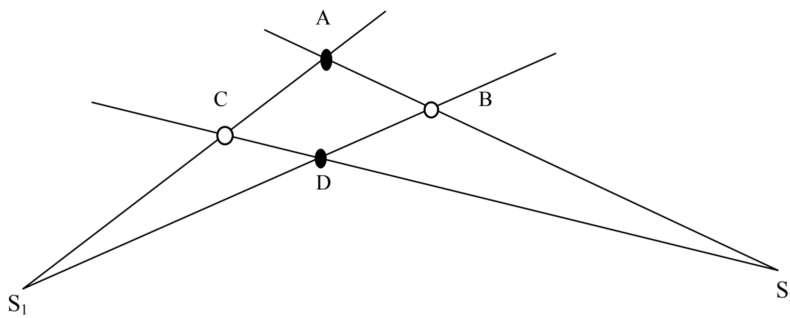


Figure 1.1: Ambiguity for Two Sensors and Two Targets

This paper focuses on the ambiguity problem. We establish bounds on the number of possible candidate targes for various sensor and target configurations. The paper is organised as follows. Section 2 details the notation and terminology used throughout the paper. Further, some basic results are established. The case when there are two sensors is considered in Section 3. The three-sensor problem is considered in Section 4. We conclude this paper in Section 5 with a discussion when there are four or more sensors. Methods for detecting and resolving ambiguity are also discussed.

2 Preliminaries

Given m sensors and n targets, it is assumed that no sensors are collocated and that their positions are known. The sensors are of the same type so that the bearing-only measurements can be directly combined. Also, every target in the problem is detected by each of the sensors. No target can lie on the same line joining any two sensors. Under these assumptions, a **candidate target** is any point that has m-lines, one from each sensor, passing through it. Let $f(m, n)$ denote the number of m-line intersections. A **ghost** is a candidate target that is not a real target. That is, it occurs when bearing-only measurements focussed at different targets intersect. This is illustrated in Figure 2.1 for the case of two sensors (S_1 and S_2) and two targets (T_1 and T_2).

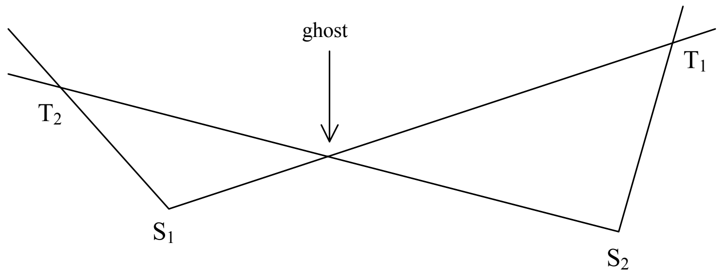


Figure 2.1: Picture of a Ghost

An ambiguous situation is illustrated in Figure 1.1 for $m = n = 2$ and in Figure 2.2 for $m = n = 3$. In the case of Figure 2.2, without prior knowledge, it is impossible to say which set, {A, E, F} or {B, C, G}, actually relates to the position of the true targets. However, it is possible to eliminate point D since selecting it contradicts the fact that there are three targets in the problem. So, with other information it may be easy to classify candidate targets as ghosts in some instances. In general, when the number of ghosts (that is black dots in Figures 1.1 and 2.2) in the problem is greater than or equal to the number of targets, an ambiguous situation will arise.

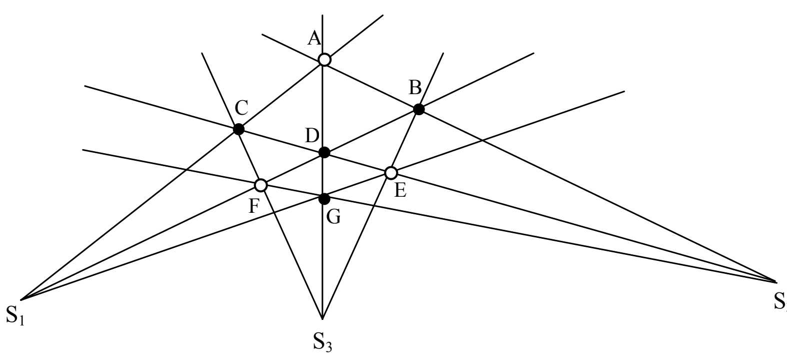


Figure 2.2: Ambiguous Situation for $m = n = 3$

As one might expect, as the number of targets increases, the occurrences of ghosts can also increase. In some cases, the number of ghosts can be more than double the number of true targets. This compounds the problem of determining which candidate targets are ghosts and which are true targets. It also increases the chance of taking a ghost into the solution since it is possible for a ghost to have a much lower cost than a true target.

When there are more sensors than targets, the problem of ghosting is eliminated as established in the following result.

Lemma 2.1: For $1 \leq n < m$, $f(m,n) = n$.

Proof: Obviously $f(n,n) \geq n$ (see Figures 2.1 and 2.2). By increasing the number of sensors ($m > n$), the true targets will now have m-line crossings and the ghosts will still have n-line crossings. Thus, ghosts no longer exist in the problem. Therefore, $f(m,n) = n$ for $m > n$. □

Consequently, an ambiguous situation can only arise when $m \leq n$. Bounds on $f(m,n)$ will help in identifying particular sensor-target configurations that have a high number of ghosts. In general, the lower bound is given in the next theorem.

Lemma 2.2: Given m sensors and n targets, $f(m,n) \geq n$ for $m \geq 2$ and $n \geq 1$.

Proof: This comes from the fact that each target is an m-line crossing and there are n targets, therefore $f(m,n) \geq n$. □

The next sections look at specific cases for $m = 2$ and $m = 3$ where $n \geq 1$ in both cases.

3 Two-Sensor Case

Consider a problem that has $m = 2$ sensors and n targets. Sensors are distinguished according to their field of view, ϕ_s. An omni-directional sensor has $\phi_s = 360°$ and a forward-looking sensor has $\phi_s = 180°$ (referenced from the horizontal).

Lemma 3.1: In the case of two omni-directional sensors and n targets,

$$f(2,n) \leq n^2.$$

Proof: Consider sensor S_i, $i = 1,2$ with its n lines $L_{i1}, L_{i2}, \ldots, L_{in}$ directed at the n targets. Observe that no two $L'_{1j}s$ or $L'_{2j}s$ can give rise to a 2-line crossing. So the only possible 2-line crossings come from lines L_{1j} and L_{2k}, $1 \leq j \leq n$ and $1 \leq k \leq n$.

Now consider line L_{2k}. It can intersect each of the lines $L_{11}, L_{12}, \ldots, L_{1n}$ and hence can contribute up to n 2-line crossings. Consequently

$$f(2,n) \leq n^2,$$

as required. □

The general lower bound in Lemma 2.2 is valid, but is weak when there are more than two targets. Now taking into account that some of the bearing lines may be parallel, a sharper bound can be developed.

In Figure 3.1 there are two sensors, S_1 and S_2, in a plane that has been divided into three regions, R_1, R_2 and R_3. In the plane there are n targets distributed such that there are a targets in R_1, b targets in R_2 and c targets in R_3. That is, $n = a + b + c$.

For this configuration the number of 2-line crossings is given by

$$f(2,n) = n^2 - ac \tag{1}$$

where n^2 comes from Lemma 5.3 and ac is subtracted because the rays from S_1 focussed at the a targets in R_1 will never intersect the rays from S_2 focussed at the c targets in R_3.

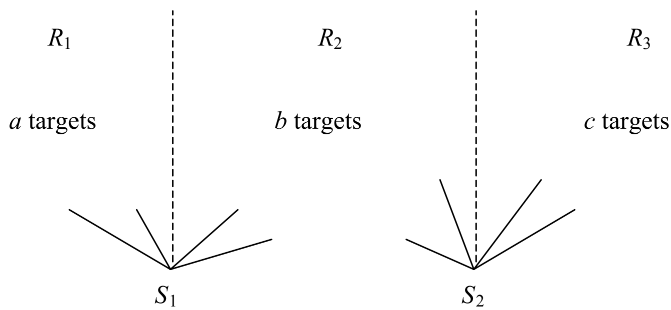


Figure 3.1: Problem Configuration

It is possible for rays from S_1 and S_2 to be parallel in R_1 and/or R_3 only. If parallel rays occur, then we have over counted the number of 2-line crossings in (1). Let i and j represent the number of targets associated with parallel rays in R_1 and R_3 respectively. This is illustrated in Figure 3.2 for R_1 where $a = 3$ and $i = 3$. Note that lines that are parallel are the same style. The over-counting due to the parallel rays is

$$i^2 - \sum_{k=1}^{i} k = i^2 - \frac{i}{2}(i+1) = \frac{i}{2}(i-1)$$

where i^2 is the maximum number of 2-line crossings for the i targets and the summation term is the actual number of 2-line crossings from the targets associated with parallel rays.

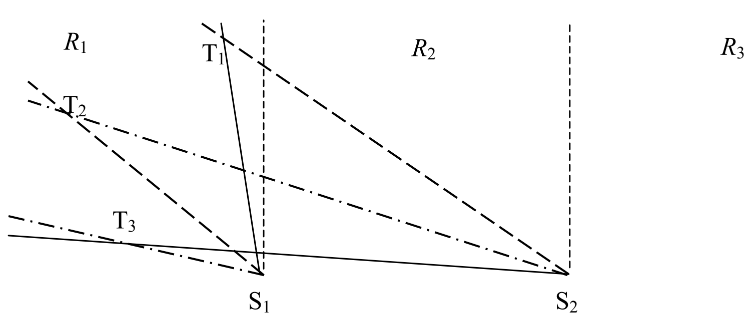


Figure 3.2: Problem Configuration with Parallel Rays

Similarly this can be applied to the j targets in R_3. Then, by subtracting the over-count from (1) it can be shown that the number of 2-line crossings is given by,

$$f(2,n) = n^2 - ac - \binom{i}{2} - \binom{j}{2}. \tag{2}$$

Lemma 3.2: In the case of two forward-looking sensors,

$$f(2,n) \geq \frac{1}{2}n(n+1). \tag{3}$$

Proof: From (2),

$$f(2,n) = n^2 - ac - \binom{i}{2} - \binom{j}{2}$$

where $a+b+c=n, \quad 0 \leq i \leq a$ and $0 \leq j \leq c$. So,

$$\begin{aligned} f(2,n) &\geq n^2 - ac - \binom{a}{2} - \binom{c}{2} \\ &= n^2 - \frac{1}{2}a(a+c-1) - \frac{1}{2}c(a+c-1) \\ &= n^2 - \frac{1}{2}(a+c)(a+c-1) \\ &= n^2 - \frac{1}{2}(n-b)(n-b-1) \\ &\geq \frac{1}{2}n(n+1). \end{aligned}$$

□

Remark: The equality in equation (3) occurs when $b=0$, $i=a$ and $j=c$. This is illustrated in Figure 3.3 for $n=6$. Here, each target is associated with at least one parallel line. Note that parallel lines have the same number of “<”.

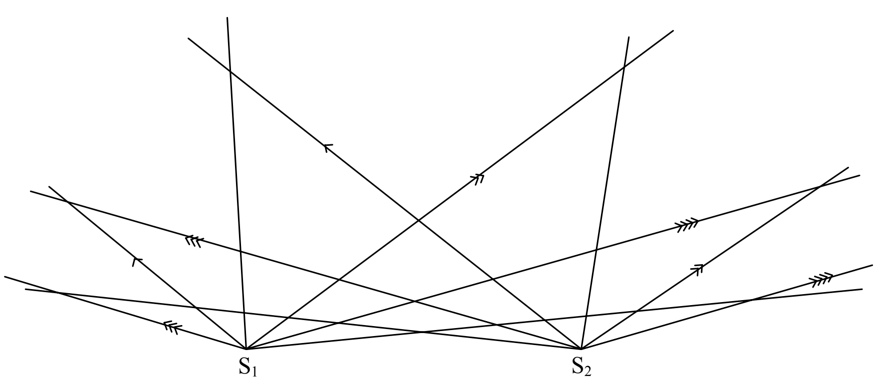


Figure 3.3: Maximum Construction for $f\,(2,6) = 21$

Lemma 3.3: Given two omni-directional sensors,

$$f(2,n) \geq \begin{cases} \frac{1}{4}n(n+2), & \text{for } n \text{ even,} \\ \frac{1}{4}(n+1)^2, & \text{for } n \text{ odd.} \end{cases}$$

Proof: Given the two sensors are omni-directional, let there be n_1 targets above and n_2 targets below the sensors respectively, such that $n = n_1 + n_2$. Once again the problem is to determine the minimum number of 2-line crossings.

If the problem is split into two parts, and the two sensors are considered to be forward-looking at the n_1 targets and the n_2 targets respectively, then the problem is the same as that in Lemma 3.2.

Hence,

$$\begin{aligned} f(2,n) &= f(2,n_1) + f(2,n_2) \\ &\geq \frac{1}{2}n_1(n_1+1) + \frac{1}{2}n_2(n_2+1) \\ &= \frac{1}{2}n(n+1) - n_1 n_2 \\ &= \frac{1}{2}n(n+1) - \frac{1}{4}(n_1+n_2)^2 + \frac{1}{4}(n_1-n_2)^2 \\ &\geq \begin{cases} \frac{1}{2}n(n+1) - \left(\frac{n}{2}\right)\left(\frac{n}{2}\right), & \text{for } n \text{ even.} \\ \frac{1}{2}n(n+1) - \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right), & \text{for } n \text{ odd.} \end{cases} \\ &= \begin{cases} \frac{1}{4}n(n+2), & \text{for } n \text{ even.} \\ \frac{1}{4}(n+1)^2, & \text{for } n \text{ odd.} \end{cases} \end{aligned}$$

as required. □

Lemmas 3.1 to 3.3 establish the following theorem.

Theorem 3.1: Given two omni-directional sensors,

$$\frac{1}{4}n(n+2) \leq f(2,n) \leq n^2,$$

for even n and

$$\frac{1}{4}(n+1)^2 \leq f(2,n) \leq n^2,$$

for odd n.

Equality in both the lower and upper bound is achievable in Theorem 3.1. Further, the configuration that gives the maximum number of crossings is unique. In general, the lower bound would be higher than the one given in Theorem 3.1 simply because it would be very unlikely in a real-world problem that the bearing lines would be parallel.

4 Three-Sensor Case

In this case, the upper bound for $m = 2$ is still valid but is not very good. A stronger bound is given in the next theorem.

Theorem 4.1: Given three forward-looking sensors

$$n \leq f(3, n) \leq \left\lfloor \frac{3}{4}n^2 + \frac{1}{4} \right\rfloor. \tag{4}$$

Proof: The lower bound comes from Lemma 2.2. For the upper bound, Lemma 3.1 states that the maximum number of 2-line crossings is n^2. Consider the two sensors first, the most number of points a line from the third sensor, S_3, can pass through is 3 then 2 then 1 (see Figure 2.2). If the lines from S_3 are placed such that they pass through the most number of points in the set of 2-line crossings, the following relationship can be established,

$$f(3, n) \leq n^2 - 2\left[1 + 2 + \ldots + \frac{n}{2}\right] + \frac{n}{2},$$

for even n and

$$f(3, n) \leq n^2 - 2\left[1 + 2 + \ldots + \left(\frac{n-1}{2}\right)\right],$$

for odd n.

These equations simplify to

$$f(3, n) \leq \frac{3}{4}n^2$$

for n even and

$$f(3, n) \leq \frac{3}{4}n^2 + \frac{1}{4},$$

for n odd, which is the required upper bound. □

We refer to a **feasible configuration** as one in which exactly n targets can be assigned to it. The upper bound in (4) is only achievable for odd n. For even n, there exists a unique sensor-target configuration that could yield equality, but it is not possible to assign $n$targets. That is, the configuration is non-feasible as illustrated in Figure 4.1 for the case when $n = 4$. Therefore, for even n, the upper bound must be restated as,

$$f(3, n) < \frac{3}{4}n^2.$$

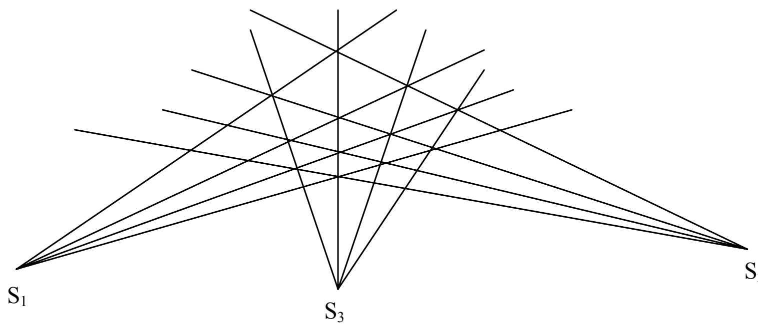


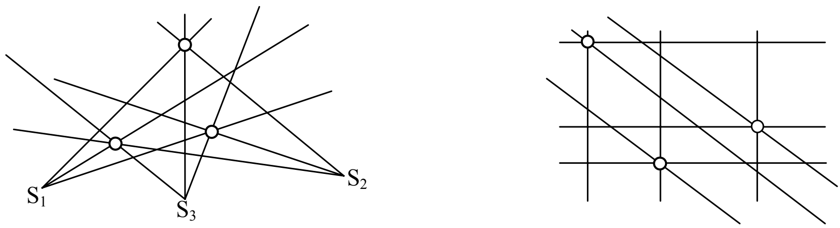
Figure 4.1: Non-Feasible Configuration for Three Sensors and Four Targets

The authors in [5] make the comment "we may virtually eliminate the ghosting problem in target position estimation by associating angle-only measurements of four or more passive sensors". It has been shown that as the number of sensors is increased, the number of ghosts is significantly reduced. This is evident in a decrease of candidate targets by approximately 25% for the three sensors case compared to the two-sensor case. However, there still exists the possibility of accepting a ghost into the solution and ambiguity such as that in Figure 2.2 occurring.

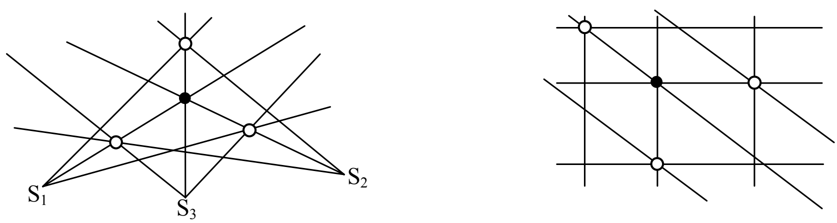
Theorems 3.1 and 4.1 give a range of possible values for $f(2, n)$ and $f(3, n)$, respectively. It is interesting to know if for a given number of targets, the range is achievable. This is the realisability problem. We investigate this problem for the three sensor case when there are three and four targets. The idea is to construct diagrams like those in Figures 2.2 and 4.1.

Consider first the case with three targets. From Theorem 4.1, the lower bound for $f(3, 3)$ is three and the upper bound is seven. The diagrams on the left in Figure 4.2 give examples whereby each number in this range is achievable. The black dots indicate a candidate target and a white dot denotes a candidate target that is declared to be an actual target.

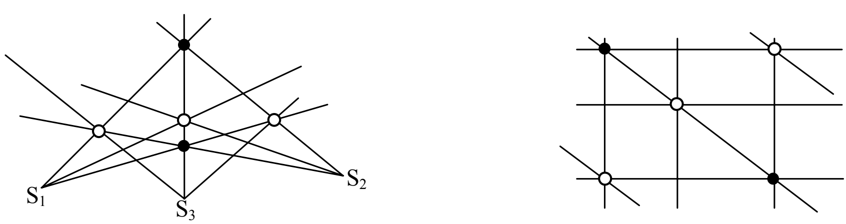
An alternate way of representing the diagrams is via a grid-like system that is given on the right of Figure 4.2. In the original diagram, the lines emanating from sensor S_1 correspond to the horizontal lines in the grid system. They are parallel to maintain the fact that only one target exists per line. Similarly, the vertical and diagonal lines map the lines from sensors S_2 and S_3 respectively. Once again, the intersection between the three lines represents a candidate target and a white dot denotes a true target. One observation is the fact that the grid does not necessarily have to be regular. However, the upper bound is only achievable with a regular grid.



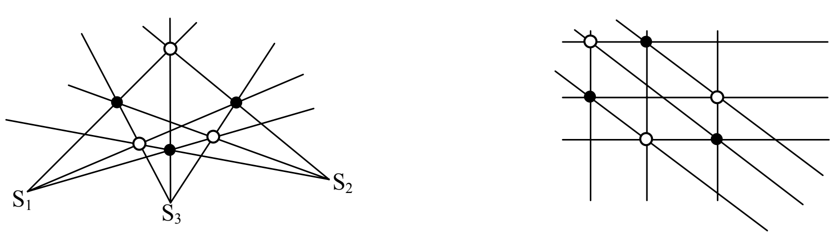
(a) $f(3,3) = 3$



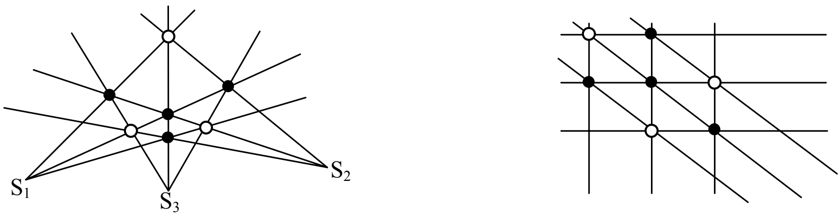
(b) $f(3,3) = 4$



(c) $f(3,3) = 5$



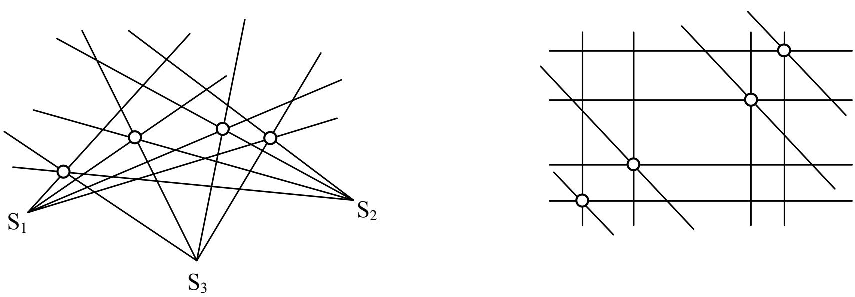
(d) $f(3,3) = 6$



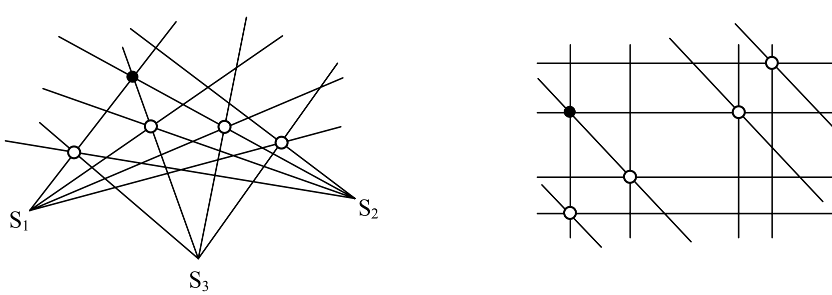
(e) $f(3,3) = 7$

Figure 4.2: Achievability of the Range for $f(3,3)$ and an Alternate Representation

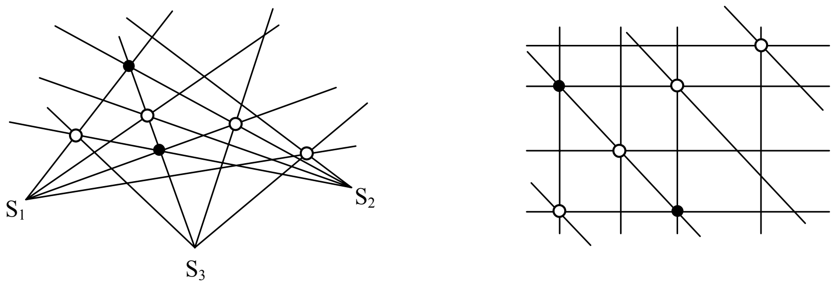
Applying this idea to the case of four targets yields the diagrams in Figure 4.3. The range of values for $f(3,4)$ is 4 to 12. Here, all values are achievable up to and including 10.



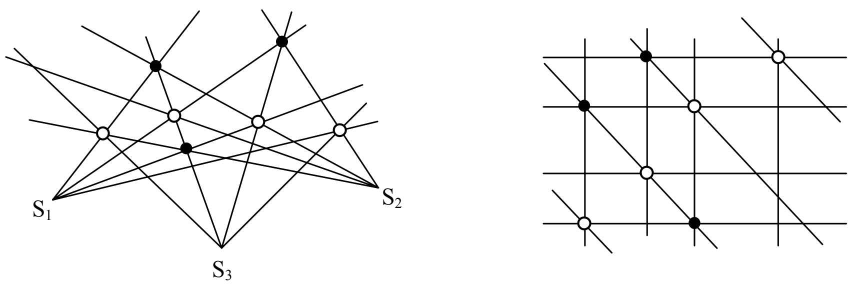
(a) $f(3,4) = 4$



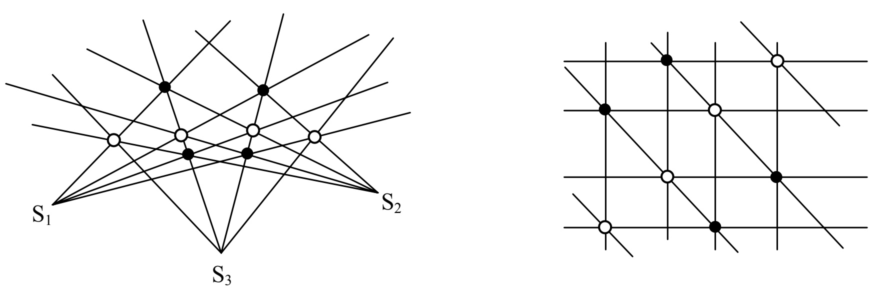
(b) $f(3,4) = 5$



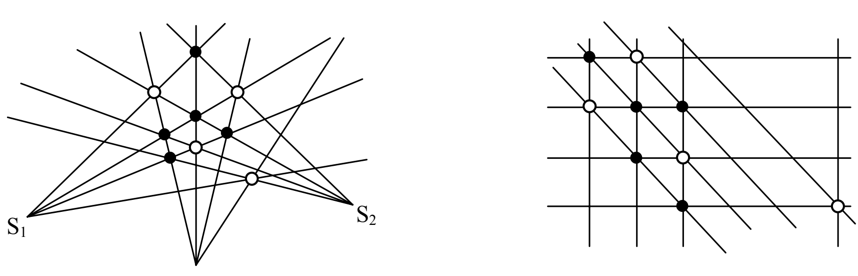
(c) $f(3,4) = 6$



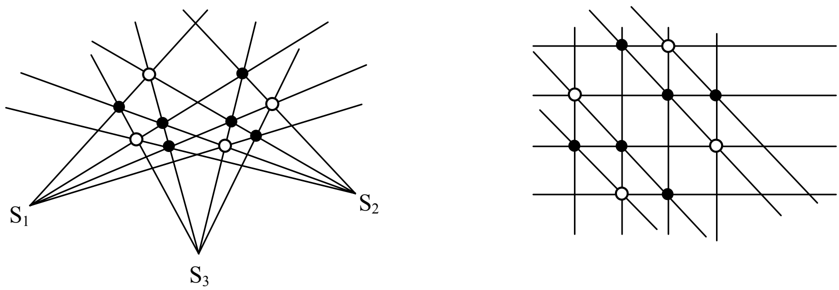
(d) $f(3,4) = 7$



(e) $f(3,4) = 8$



(f) $f(3,4) = 9$



(g) $f(3,4) = 10$

Figure 4.3: Achievability of the Range for $f(3,4)$ and an Alternate Representation

The configurations that give exactly 11 and 12 candidate targets are displayed in Figures 4.4 and 4.5, respectively. In each case it is impossible to choose four targets from the candidate targets without violating the fact that each line must have only one target assigned to it. The verification of this involves routine checking. We illustrate this with one example.

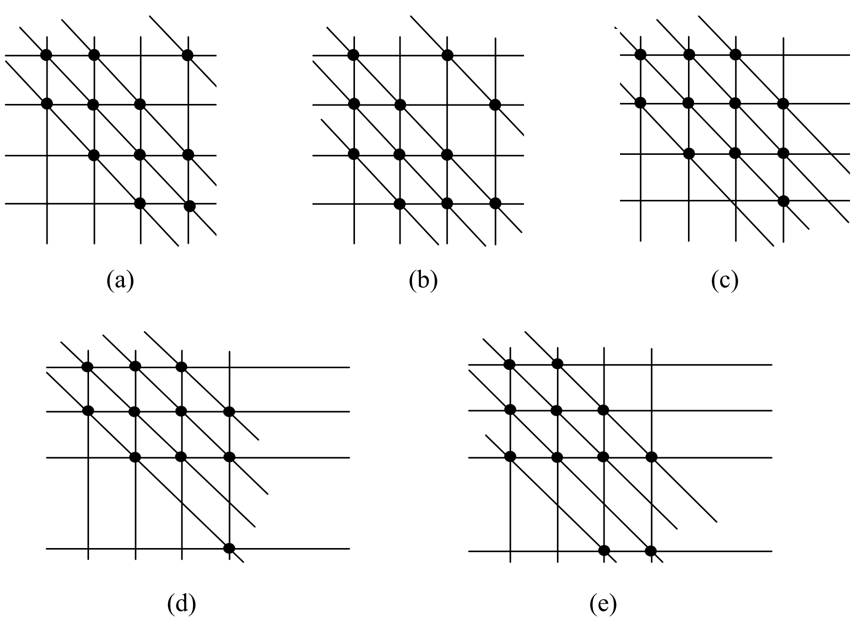
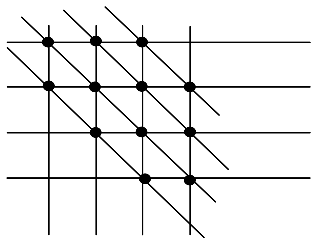
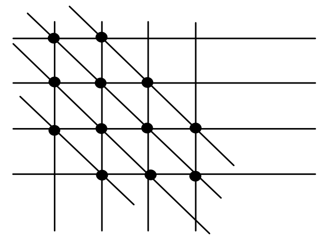


Figure 4.4: Non-feasible Configurations for $f(3,4) = 11$





(a) (b)

Figure 4.5: Non-feasible Configurations for $f(3,4) = 12$

Consider the configuration displayed in Figure 4.5(a). We assign labels to the points (candidate targets) as below (Figure 4.6).

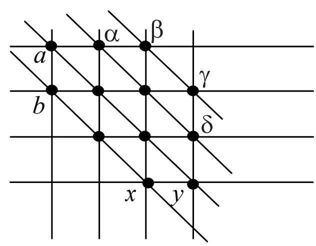


Figure 4.6: Labelled Non-feasible Configuration for $f(3,4) = 12$

Let T be a feasible set of targets. Consider points a and b (x and y) that have exactly one vertical (horizontal) line passing through them. Observe that either a and $x \in T$ or b and $y \in T$. Now the only points that are not on a line passing through a and x (b and y) are γ and δ (α and β). As each of γ and δ and α and β are on the same line, the maximum number of targets that can be in T are 3. Hence the configuration of Figure 4.5(a) is non-feasible.

Using a similar argument (but with more cases to consider) we can establish that $f(3,6) < 27$. Thus equality in equation (4) is not achievable for $n = 4$ and 6. We conjecture that for all even n,

$$f(3,n) < \frac{3}{4}n^2.$$

In fact, we believe that

$$f(3,n) \leq \frac{3}{4}n^2 - \frac{n}{2}$$

for even n.

We have shown that equality is achievable for $n = 3$ and can illustrate it for $n = 5$ and 7. We believe the upper bound in equation (4) is achievable for odd n.

Conjecture 1: Given three sensors and n targets, where n is even,

$$f(3,n) \leq \left(\frac{3}{4}n^2 - \frac{n}{2}\right).$$

For Conjecture 1, a graph theoretical formulation of the feasibility problem can be obtained as follows. We consider each candidate target as a vertex and we join two vertices by an edge if there is a line passing through them in the grid representation. We call the resulting graph $G(n)$. For feasibility, we need to find a set of n vertices no two of which are adjacent, that is, an independent set of n vertices in $G(n)$. In graph theoretic terminology the problem is to show that the "independence number" of the graph is n. This is, for general graphs, a difficult problem.

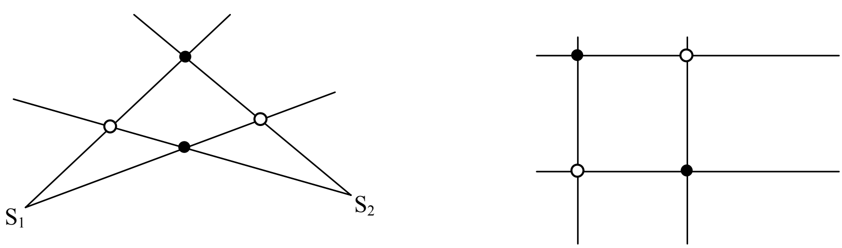
5 Discussion

From Lemma 2.1 we know that the first occurrence of ambiguity is when $m = n$. Also, the upper bound in equation (4) is a valid bound for $m \geq 4$ as well. For general n, we make the following conjecture:

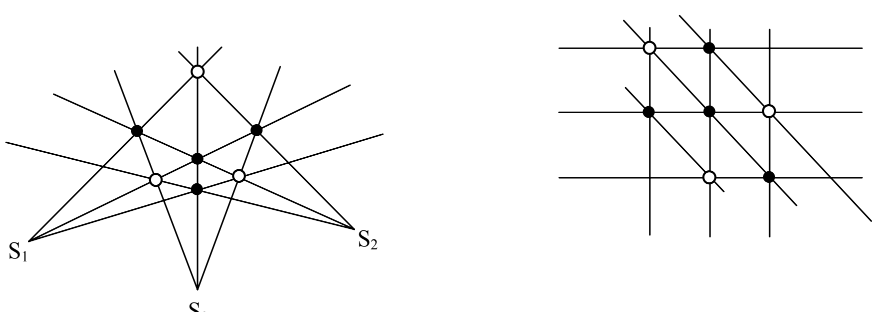
Conjecture 2: Given n sensors and n targets,

$$f(n,n) \leq \begin{cases} 2n+1, & \text{for odd } n, \\ 2n, & \text{otherwise.} \end{cases} \tag{5}$$

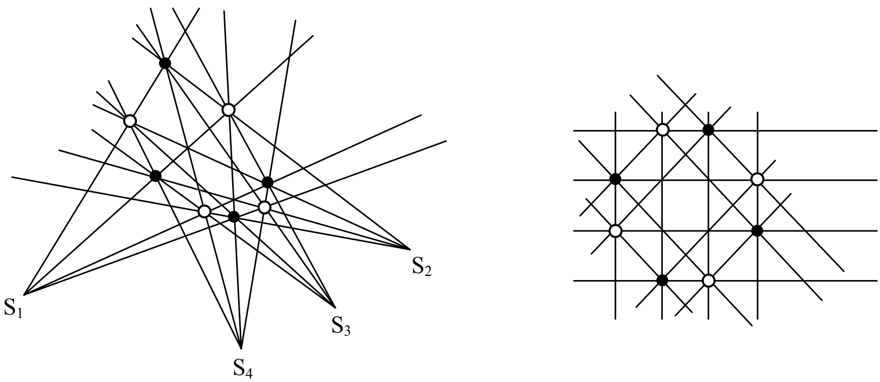
Constructions for the cases $n = 2$, 3 and 4 are given in Figure 5.1 and the results in terms of $f(n,n)$ are given in Table 5.1.



(a) $f(2,2) = 4$



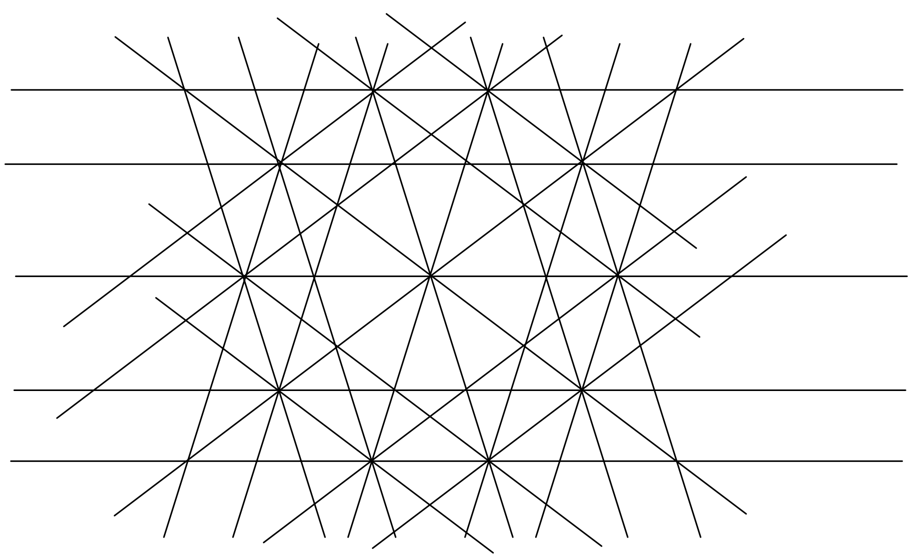
(b) $f(3,3) = 7$



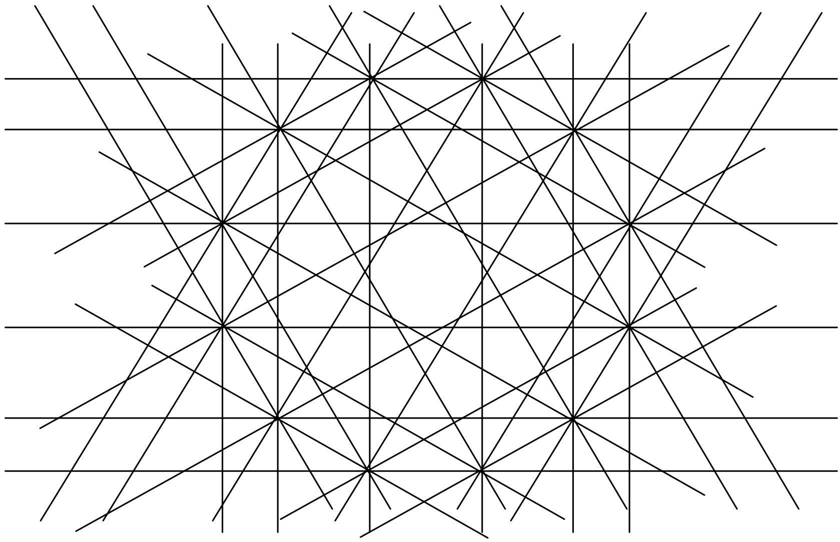
(c) $f(4,4) = 8$

Figure 5.1: $f(n,n)$ for $n = 2$, 3 and 4.

As the number of sensors and targets increase, the diagrams become much harder to draw; for this reason, only the grid-like representation described in the previous section is given for $n = 5$ and 6 in Figure 5.2.



(a) $f(5,5) = 11$



(b) $f(6,6) = 12$

Figure 5.2: $f(n,n)$ for $n = 5$ and 6.

For the ambiguity to occur, each bearing line emanating from the sensors has a true target and a ghost on it. In the odd case, the extra ghost comes from the intersection of the bearing lines crossing in the centre. A summary of the cases is given in Table 5.1. The values in the table are the same as those given by the conjecture in equation (5) giving support to our claim.

n	$f(n,n)$
2	4
3	7
4	8
5	11
6	12

Table 5.1: Values of $f(n,n)$ for varying n

The multisensor data association problem can be formulated as a three-dimensional assignment problem that can be solved in a variety of ways. The solution of the assignment problem gives the number of targets and their corresponding positions. It also says which measurements are false alarms. However, current algorithms do not address the problem of ambiguity that was discussed in previous sections. An implication of the ambiguity is that current algorithms may converge to a mathematically correct solution, but this solution is not physically valid. Thus the effect of ambiguity is the introduction of local minima into the solution space.

There are several ways of detecting ambiguity. One would be to consider the value of the k-best solutions to the assignment problem. If two or more solutions had very similar costs, this would indicate the presence of ambiguity. This is easily done in a Branch and Bound algorithm by replacing the weak inequalities in the fathoming test with strict inequalities and storing each of the solutions. At the conclusion of the Branch and Bound algorithm, all the stored solutions will be the optimal solutions.

The solution to the assignment problem is $\hat{x}_{ijk}$ for all i, j and k. By considering a subset of the solution set where $\hat{x}_{ijk} = 1$, we can devise a brute force method to generate the next best solution. This is explained via an example. Consider the problem as given in Figure 5.3 where all targets are detected and there are no false alarms. We know there are two solutions, γ_1 and γ_2 respectively, both with the same optimal value of $z_{optimal}$. The first solution is

$$\gamma_1 = \{A, C, E\} = \{Z_{ijk} : \hat{x}_{ijk} = 1\} = \{Z_{312}, Z_{121}, Z_{233}\}$$

and the second

$$\gamma_2 = \{B, D, F\} = \{Z_{ijk} : \hat{x}_{ijk} = 1\} = \{Z_{211}, Z_{132}, Z_{323}\}.$$

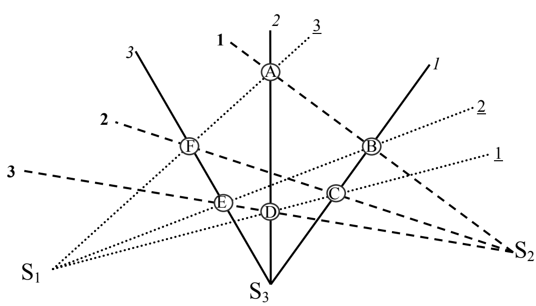


Figure 5.3: Example of Ambiguity for Three Sensors and Three Targets Used to Illustrate the Brute Force Method of Finding the k-Best Solution

Assuming that the algorithm converged to the first solution, if we set all of the variables in γ_1 to zero and resolved the problem with these variables fixed, we would obtain another solution, z_{soln2}. In this instance because γ_1 and γ_2 are mutually exclusive we would have $z_{\text{soln2}} = z_{optimal}$ or $z_{\text{soln2}} \approx z_{optimal}$. This would alert us to the presence of ambiguity. In the non-ideal situations where there are missed detections and false alarms and ambiguity present, we could have a situation where some of the variables where $\hat{x}_{ijk} = 1$ may appear in both solution sets so it would be foolish to fix all the variables that equal one to zero. To check for ambiguity in this instance it would be more realistic to fix a variety of the variables to zero and resolve the problem. It would be advisable to do this for a variety of combinations.

It has also been observed that for $m = n = 3$ (see Figure 2.2), $m = n = 4$ and $m = n = 5$ an alternating pattern between true targets and the ghosts. A method for moving between one solution set to the other could be devised. When there are more sensors than targets, ambiguity does not occur. So, one way of resolving ambiguity could be to add additional sensors to the problem. In practice this is not always possible and it will also increase the complexity of the problem to solve. Alternatively, other information such as frequency could be used to resolve ambiguity.

References

[1] S.M. Andrijich and L. Caccetta, *Modelling Multisensor Data Association Problems*. Proceedings International Congress on Modelling and Simulation (L. Oxley et al. Editors), Published by Modelling and Simulation Society of Australia and New Zealand, Inc., Vol. 4 (1999), 1027–1032.

[2] S.M. Andrijich and L. Caccetta, *Modelling Multisensor Data Association Problems*. Mathematical and Computer Modelling (2000) (in press).

[3] S.M. Andrijich and L. Caccetta, *Solving the Multisensor Data Association Problem*. Nonlinear Analysis, Vol. 47, No. 8 (2001), 5525–5536.

[4] E. Balas and M.J. Saltzman, *An Algorithm for the Three-Index Assignment Problem*. Operations Research, Vol. 39, No. 1 (1991), 150–161.

[5] S. Deb, K.R. Pattipati and Y. Bar-Shalom, *A Multisensor-Multitarget Data Association Algorithm for Heterogeneous Sensors*. IEEE Transactions on Aerospace and Electronic Systems, Vol. 29, No. 2 (1993), 560–568.

[6] S. Deb, M. Yeddanapudi, K. Pattipati and Y. Bar-Shalom, *A Generalised S-D Assignment Algorithm for Multisensor-Multitarget State Estimation*. IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 2 (1997), 523–538.

[7] A.M. Frieze and J. Yadegar, An Algorithm for Solving 3-Dimensional Assignment Problems with Application to Scheduling a Teaching Practice. Journal of Operational Research Society, Vol. 32 (1981), 989–995.

[8] D.L. Hall and J. Llinas, *An Introduction to Multisensor Data Fusion*. Proceedings of the IEEE, Vol. 85, No. 1 (1997), 6–23.

[9] R.A. Iltis and R.L. Anderson, *A Consistent Estimation Criterion for Multisensor Bearings-Only Tracking*. IEEE Transactions on Aerospace and Electronic Systems, Vol. 32, No. 1 (1996), 108–120.

[10] D. Magos, *Tabu Search for the Planar Three-Index Assignment Problem*. Journal Global Optimization, Vol. 8, No. 1 (1996), 35–48.

[11] K.R. Pattipati, S. Deb, Y. Bar-Shalom and R.B. Washburn, *A New Relaxation Algorithm and Passive Sensor Data Association*. IEEE Transactions on Automatic Control, Vol. 37, No. 2 (1992), 198–213.

[12] A.B. Poore, *Multidimensional Assignment Formulation of Data Association Problems Arising From Multitarget and Multisensor Tracking*. Computational Optimization and Applications, Vol. 3 (1994), 27–57.

[13] A.B. Poore, *Multidimensional Assignments and Multitarget Tracking*. DIMACS Series Discrete Mathematics and Theoretical Computer Science, Vol. 19 (1995), 169–198.

[14] A.B. Poore and A.J. Robertson III, *A New Lagrangian Relaxation Based Algorithm for a Class of Multidimensional Assignment Problems*. Computational Optimization and Applications, Vol. 8, No. 2 (1997), 129–150.

[15] P.K. Varshney, *Scanning the Special Issue on Data Fusion*. Proceedings of the IEEE, Vol. 85, No. 1 (1997), 3–5.

[16] E. Waltz and J. Llinas, *Multisensor Data Fusion*. Artech House, Inc., USA (1990).

[17] M. Yeddanapudi, Y. Bar-Shalom and K.R. Pattipati, *IMM Estimation for Multitarget-Multisensor Air Traffic Surveillance*. Proceedings of the IEEE, Vol. 85, No. 1 (1997), 80–94.

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