Disjoint Hamiltonian Cycles in Graphs*

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Abstract

Let G be a $2(k+1)$-connected graph of order n. It is proved that if $uv \notin E(G)$ implies that $\max\{d(u), d(v)\} \geq \frac{n}{2} + 2k$ then G contains $k+1$ pairwise disjoint Hamiltonian cycles when $\delta(G) \geq 4k+3$.

1. Introduction

All graphs we consider are finite and simple. We use standard terminology and notation from Bondy and Murty [2] except as indicated. Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. For a subset U of $V(G)$, $G[U]$ is the subgraph of G induced by U. For two disjoint subsets (resp. subgraphs) S, T of $V(G)$ (resp. G), put

$$E(S,T) = \{st \in E(G) |\, s \in S, t \in T\},$$
$$\bar{E}(S,T) = \{st \notin E(G) |\, s \in S, t \in T\},$$
$$N_T(S) = \{t \in T |\, t \text{ is adjacent to some vertex in } S\},$$
$$\bar{N}_T(S) = \{t \in T |\, t \text{ is not adjacent to any vertex in } S\},$$
$$d_T(S) = |N_T(S)|, \qquad \bar{d}_T(S) = |\bar{N}_T(S)|;$$

when $S = \{s\}$, we write $d_T(s)$ and $\bar{d}_T(s)$ for $d_T(\{s\})$ and $\bar{d}_T(\{s\})$. Let $P = uv \cdots w$ and $Q = xy \cdots z$ be two vertex-disjoint paths of G. If ux and wz are the edges of G, we denote by PQ the cycle $P \cup Q \cup \{ux, wz\}$ with a given orientation in the order from x to z along the path Q. For a cycle C with an given orientation and a vertex $v \in V(C)$, we denote by v_C^- and v_C^+ the predecessor and successor of v on C, respectively. Two Hamiltonian cycles are called disjoint when they share no common edge, and similar terminology will be applied to disjoint paths.

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Let x be a real number. We denote by $[x]$ the maximum integer less than or equal to x.

The following theorem due to Geng-hua Fan [3] is well known.

Theorem A If a 2-connected graph G of order n satisfies the condition $d(u,v) = 2 \Longrightarrow max\{d(u), d(v)\} \geq \frac{n}{2}$, then G contains a Hamiltonian cycle.

The proof of Fan's result was simplified by F.Tian [5]. In 1993, S.Zhou [6] proved the following theorem by using the method essentially same as used by Tian.

Theorem B If a 4-connected graph G of order n satifies the condition $d(u,v) = 2 \Longrightarrow max\{d(u), d(v)\} \geq \frac{n}{2} + 2$, then G contains 2 Hamiltonian cycles.

On disjoint Hamiltonian cycles, H.Li [4] proved in 1989 the following interesting result.

Theorem C Let n and k be positive integers such that $n \geq 8k^2 - 5$, and let G be a graph on n vertices with minimum degree δ satisfying $2k+1 \leq \delta \leq 2k+2$. If $d_G(u) + d_G(v) \geq n$ for any pair of nonadjacent vertices u and v, and if $l_1, \cdots, l_k$ are integers satisfying $3 \leq l_1 \leq l_2 \cdots \leq l_k \leq n$, then G contains k disjoint cycles of length $l_1, l_2, \cdots, l_k$, respectively. In particular, under these conditions G contains k disjoint Hamiltonian cycles.

There are many results on Hamiltonian cycles, but few on disjoint Hamiltonian cycles. Here we focus our attention on the study of disjoint Hamiltonian cycles in graphs. As in [6], for a nonnegative integer k, a graph of order n is called a Fan $2k$-type graph if $d(u,v) = 2$ implies $max\{d(u), d(v)\} \geq \frac{n}{2} + 2k$. In this paper, we call a graph of order n an Ore $2k$-type graph if $uv \notin E(G)$ implies $max\{d(u), d(v)\} \geq \frac{n}{2} + 2k$. We will prove the following Theorem.

Theorem 1 Let G be a $2(k+1)$-connected Ore $2k$-type graph. If $\delta(G) \geq 4k+3$ then G contains $k+1$ disjoint Hamiltonian cycles.

We surmise the condition $\delta(G) \geq 4k+3$ can be deleted, but this task is formidable. So we pose the following conjecture.

Conjecture 1 For any nonnegative integer k, every $2(k+1)$-connected Ore $2k$-type graph contains $k+1$ disjoint Hamiltonian cycles.

It is easy to see that the proof of Conjecture 1 will be a stepping stone in the proof of the following conjecture 2 posed by S.Zhou in [6].

Conjecture 2 For any nonnegative integer k, every $2(k+1)$-connected Fan $2k$-type graph contains $k+1$ disjoint Hamiltonian cycles.

We will prove Theorem 1 in section 2. As an application of the method established in section 2, we will give, in section 3, an alternative proof of Conjecture 1 for $k = 1$. We attempt to explain how the method established in section 2 might be useful in proving the conjecture 1.

2. Proof of Theorem 1

In this section, all graphs we consider are $2(k+1)$-connected Ore $2k$-type. The following Lemmas are useful in proving our main results.

Lemma 1 If G is a Ore $2k$-type graph of order n, and u, v are nonadjacent vertices of G which satisfy $\min\{d(u), d(v)\} \geq \frac{n}{2} + 2k$, then

(1) $G + uv$ is also a Ore $2k$-type graph, and

(2) G contains $k+1$ disjoint Hamiltonian cycles if and only if $G + uv$ contains $k+1$ disjoint Hamiltonian cycles.

Proof. Let x and y be nonadjacent vertices in $G' = G + uv$. Then $\max\{d_{G'}(x), d_{G'}(y)\} \geq \max\{d_G(x), d_G(y)\} \geq \frac{n}{2} + 2k$, so (1) is valid. For (2), let $C_1, C_2, \cdots, C_{k+1}$ be $k+1$ disjoint Hamiltonian cycles of $G + uv$. We will prove that G contains $k+1$ disjoint Hamiltonian cycles as well. If $uv \notin \bigcup_{i=1}^{k+1} E(C_i)$, then $C_1, C_2, \cdots, C_{k+1}$ are the required cycles of G; otherwise, say $uv \in E(C_1)$, then $G' = G - \bigcup_{i=2}^{k+1} E(C_i)$ contains a Hamiltonian path $C_1 - uv = x_1(=u)x_2 \cdots x_n(=v)$. Let

$$M = \{x_i | x_1 x_i \in E(G'), 2 \leq i \leq n-1\},$$

$$N = \{x_i | x_{i-1} x_n \in E(G'), 3 \leq i \leq n\}.$$

Since $x_1 \notin M \cup N$, we get that $|M \cup N| \leq n - 1$. On the other hand, the inequality $\min\{d(u), d(v)\} \geq \frac{n}{2} + 2k$ implies that $\min\{d_{G'}(u), d_{G'}(v)\} \geq \frac{n}{2}$. Hence we get that $|M| + |N| = d_{G'}(x_1) + d_{G'}(x_n) \geq n$, and that $|M \cap N| = |M| + |N| - |M \cup N| \geq 1$.

Therefore there is a vertex $x_i \in M \cap N$, and so G' has a Hamiltonian cycle $C_1' = x_i x_1 x_2 \cdots x_{i-1} x_n x_{n-1} \cdots x_{i+1} x_i$ disjoint from the cycles $C_2, \cdots, C_{k+1}$. Lemma 1 is proved.

Lemma 2 Let G be a complete graph of order n. Then G contains $[\frac{n}{2}]$ disjoint Hamiltonian paths.

Proof. Let the vertices of G be $v_1, v_2, \cdots, v_n$. Then the $[\frac{n}{2}]$ Hamiltonian paths required are

$$v_{1+i}v_{p+i}v_{2+i}v_{p+i-1}\cdots v_{j+i+1}v_{p+i-j}\cdots v_{[\frac{n}{2}]+i+2}v_{[\frac{n}{2}]+i+1}, \text{ if } n \text{ is odd},$$

$$v_{1+i}v_{p+i}v_{2+i}v_{p+i-1}\cdots v_{j+i+1}v_{p+i-j}\cdots v_{\frac{n}{2}+i}v_{\frac{n}{2}+i+1}, \text{ if } n \text{ is even}$$

for $i = 1, 2, \cdots, [\frac{n}{2}]$, where the subscripts are all taken modulo n.

Lemma 3 Let G be a complete graph of order n. Then G contains $[\frac{n-1}{2}]$ disjoint Hamiltonian cycles.

Proof. Let $u_0 \in V(G)$ be a vertex of G. By Lemma 2, $G - \{u_0\}$ contains $[\frac{n-1}{2}]$ disjoint Hamiltonian paths. Then the Hamiltonian cycles required are obtained from these paths by joining the vertex u_0 to each end of these paths.

Now we start the proof of Theorem 1.

Let G be a complete graph of at least $2k$ vertices, and $(u_1, v_1), \cdots, (u_k, v_k)$ be k pairs of vertices of G. By the proof of Lemma 2, we see that G has k disjoint

Hamiltonian paths $P(u_i, v_i), i = 1, 2, \cdots, k$, with u_i and v_i, $i = 1, 2, \cdots, k$, as their endvertices.

Since G is $2(k+1)$-connected, G has at least $2k+3$ vertices. By Lemma 3, we only need to consider the case that G is not complete, and let

$$S = \{u \in V(G) | d(u) \geq \frac{n}{2} + 2k\}.$$

Then $S \neq \emptyset$ since G is not complete. By Lemma 1, we may assume that $G[S]$ is complete. By Lemma 3, we further assume that $S \neq V(G)$. Let $G_i = (V_i, E_i)$ $(1 \leq i \leq \omega)$ denote the components of $G[V \setminus S]$, and let

$$S_i = N_S(V_i), \ \ 1 \leq i \leq \omega.$$

By the hypothesis that G is an Ore $2k$-type graph, we have

Lemma 4 $\omega = 1$ and G_1 is complete.

Lemma 5 $|S_1| \geq 2(k+1)$.

Proof. Suppose to the contrary that $|S_1| < 2(k+1)$. In view of the fact that G is $2(k+1)$-connected, we have that $S = S_1$ since otherwise S_1 forms an cutset of G.

Since G is not complete, there is a vertex, say $s \in S$, which is not adjacent to some vertex in V_1. Since $s \in S$, we have $d_G(s) \geq \frac{n}{2} + 2k$, and then

$$|S| \geq \frac{n}{2} + 2k + 1 - (|V_1| - 1) = \frac{n}{2} + 2k - |V_1| + 2,$$

while $|S| + |V_1| = n$, we get $n \geq 4k + 4$.

On the other hand, because of $|S| \leq 2k + 1$, we get

$$|V_1| = n - |S| \geq 4k + 4 - (2k+1) = 2k + 3 \geq |S| + 2.$$

Since $\sum\limits_{v \in V_1} d_S(v) = \sum\limits_{s \in S} d_{V_1}(s)$, we get $|V_1| \max\limits_{v \in V_1} d_S(v) \geq |S| \min\limits_{s \in S} d_{V_1}(s)$. It follows from $\frac{|S|}{|V_1|} - 1 < 0$ and $\min_{s \in S} d_{V_1}(s) \leq |V_1|$ that

$$\begin{aligned}
&\max_{v \in V_1} d_S(v) \geq \min_{s \in S} d_{V_1}(s) + (\tfrac{|S|}{|V_1|} - 1) \min_{s \in S} d_{V_1}(s) \\
&\geq \min_{s \in S} d_{V_1}(s) + (\tfrac{|S|}{|V_1|} - 1)|V_1| = \min_{s \in S} d_{V_1}(s) + |S| - |V_1|,
\end{aligned}$$

but then

$$\begin{aligned}
&\tfrac{n}{2} + 2k > \max_{v \in V_1} d_G(v) = |V_1| - 1 + \max_{v \in V_1} d_S(v) \\
&\geq |V_1| - 1 + \min_{s \in S} d_{V_1}(s) + |S| - |V_1| = |S| - 1 + \min_{s \in S} d_{V_1}(s) \\
&= \min_{s \in S} d_G(s) \geq \tfrac{n}{2} + 2k,
\end{aligned}$$

a contradiction. Lemma 5 is proved.

If $|V_1| \geq 2(k+1)$, then, by Lemma 5 and since G is $2(k+1)$-connected, there are at least $2(k+1)$ independent edges between S_1 and V_1, and by Lemma 2, G has $k+1$ disjoint Hamiltonian cycles since both $G[S]$ and $G[V_1]$ contain $k+1$ disjoint Hamiltonian paths respectively, and so we assume in the rest of the proof that $|V_1| \leq 2k + 1$. Put

$$l = \max_{s \in S}\{\bar{d}_{G_1}(s)\}.$$

Lemma 6 $n \geq 4k + 2l + 2$.

Proof. Let $s \in S$ be a vertex of G with $\bar{d}_{G_1}(s) = l$. Then $d_G(s) = n - l - 1$, together with the inequality $d_G(s) \geq \frac{n}{2} + 2k$, we get $n \geq 4k + 2l + 2$.

Denote by B_1 the bipartite graph $(S_1 \cup V_1, E(S_1, V_1))$, and by M_1 the maximum matching of B_1, and by U_1 the set of vertices of M_1 which are in S_1. Since G is $2(k+1)$-connected and $|V_1| < 2(k+1)$, we have that $|V_1| = |U_1|$. By Lemma 5, we can choose $W_1 \subseteq S_1 \setminus U_1$ such that $|U_1 \cup W_1| = |V_1 \cup W_1| = 2(k+1)$. By Lemma 6, we have that $|S \setminus W_1| = n - |V_1 \cup W_1| \geq 4k + 2l + 2 - 2(k+1) = 2k + 2l \geq 2k + 2$ since G is not complete.

Let G^* be the graph obtained from G by adding all edges between V_1 and W_1 which are not in $E(V_1, W_1)$ to G such that $G^*[V_1 \cup W_1]$ is complete.

We now consider the induced subgraphs $G^*[S \setminus W_1]$ and $G^*[V_1 \cup W_1]$. By Lemma 2, $G^*[S \setminus W_1]$ contains $k+1$ disjoint Hamiltonian paths $P_1, \cdots, P_{k+1}$; and $G^*[V_1 \cup W_1]$ contains $k+1$ disjoint Hamiltonian paths $Q_1, \cdots, Q_{k+1}$, such that P_i has endvertices either $u_i, u_i' \in U_1$, or $u_i \in U_1, s \in S \setminus (U_1 \cup W_1)$, or $s, s' \in S \setminus (U_1 \cup W_1)$; and to which correspond, Q_i has endvertices either $v_i, v_i' \in V_1$, or $v_i \in V_1, w \in W_1$, or $w, w' \in W_1$, where $u_iv_i, u_i'v_i' \in M_1$. Let $C_i = P_iQ_i, i = 1, \cdots, k+1$. Then $C_1, \cdots, C_{k+1}$ are $k+1$ disjoint Hamiltonian cycles of G^*.

To prove Theorem 1, we need to construct $k+1$ disjoint Hamiltonian cycles of G from the disjoint Hamiltonian cycles of G^*. Let $E_i = \{vw \in E(Q_i) | vw \notin E(G)\}, i = 1, 2, \cdots, k+1$. Then $|E_i| \leq 2k+1, i = 1, 2, \cdots, k+1$. Since $\delta(G) \geq 4k+3$, we have that $d_{P_i}(v) \geq 2k + \bar{d}_{W_1}(v) + 2$ for each $v \in V_1$. Consequently, we can choose $k+1$ pairwise edge-disjoint subsets $F_1, F_2, \cdots, F_{k+1}$ of G such that

1) $F_i = \{vs \in E(G) \setminus M_1 | v \in V_1, s \in S \setminus W_1$, and there is a vertex w such that $vw \in E(C_i) \setminus E(G)\}$, $i = 1, 2, \cdots, k+1$;

2) $|F_i| = |E_i|, i = 1, 2, \cdots, k+1$;

3) Every two edges in F_i have no vertex of P_i in common.

4) There is at most one edge in F_i connecting an endvertex u of P_i and a vertex $v \in V_1$ such that if $vv_{C_i}^+$ is not an edge of G then $u_{C_i}^+ \notin V_1$, and if $vv_{C_i}^-$ is not an edge of G then $u_{C_i}^- \notin V_1$.

Let $vs \in F_i$. When $vv_{C_i}^+ \notin E(G)$, we have that $v_{C_i}^+ s_{C_i}^+ \in E(G)$ because $v_{C_i}^+ \in W_1$. While $v_{C_i}^- v \notin E(G)$, we have that $v_{C_i}^- s_{C_i}^- \in E(G)$ because $v_{C_i}^- \in W_1$.

From the argument above, we can get $k+1$ Hamiltonian cycles of G, which may have edges in common, from the $k+1$ Hamiltonian cycles of G^*.

To begin with, for every integer $i (1 \leq i \leq k+1)$, we first choose $vs \in F_i$ such that s is an endvertex of P_i if possible.

If $vv_{C_i}^+ \notin E(G)$ then $v_{C_i}^+ s_{C_i}^+ \in E(G)$ since $v_{C_i}^+$ and $s_{C_i}^+$ are vertices of S, and the edge $v_{C_i}^+ s_{C_i}^+$ is said to be A-type. If $vv_{C_i}^- \notin E(G)$ then $v_{C_i}^- s_{C_i}^- \in E(G)$ since $v_{C_i}^-$ and $s_{C_i}^-$ are vertices of S, and the edge $v_{C_i}^- s_{C_i}^-$ is said to be A-type.

Replacing $vv_{C_i}^+$ and $ss_{C_i}^+$ by vs and $v_{C_i}^+ s_{C_i}^+$, or $vv_{C_i}^-$ and $ss_{C_i}^-$ by vs and $v_{C_i}^- s_{C_i}^-$, we get a Hamiltonian cycle C_i' of G^* with at least one more edge of G than C_i has.

Writing C_i for C_i', and repeating the procedure above until all edges of F_i are included in the edges of C_i, we then obtained a Hamiltonian cycle C_i of G.

It is easy to see that every A-type edge has its endvertices in S.

Using the procedure above on all cycles of G^*, we get $k+1$ cycles, still written $C_1, C_2, \cdots, C_{k+1}$, of G. All A-type edges have their endvertices in S. Clearly, if an edge is a common edge of a number of cycles, then the edge must be A-type, and so the endvertices of it are in S.

Denote by $m_e(C_1, \cdots, C_{k+1})$ the number of cycles having the edge e in common. Put

$$m(C_1, \cdots, C_{k+1}) = \sum_{e \in E(G)} \max\{m_e(C_1, \cdots, C_{k+1}) - 1, 0\}.$$

If $m(C_1, \cdots, C_{k+1}) = 0$, then $C_1, \cdots, C_{k+1}$ are the required Hamiltonian cycles of G. Suppose $m(C_1, \cdots, C_{k+1}) > 0$.

Let $uv \in E(C_i)$ be an edge belonging to two or more Hamiltonian cycles of G. Since both u and v are vertices of S, they have degree in G at least $n-l-1$. Let $G' = G - \bigcup_{j=1}^{k+1} E(C_j) + E(C_i) \setminus \{uv\}$. Then, by Lemma 6, we get

$$d_{G'}(u) + d_{G'}(v) \geq 2(n-l-1-2k) \geq 2(\frac{n}{2} + 2k + l + 1 - l - 1 - 2k) = n.$$

By the same method as that of proving Lemma 1(2), we conclude that G' has a Hamiltonian cycle C_i'. It follows that $m(C_1, \cdots, C_i', \cdots, C_{k+1}) = m(C_1, \cdots, C_{k+1}) - 1$.

Writing C_i' for C_i, and repeating this procedure until $m(C_1, \cdots, C_{k+1}) = 0$, we get $k+1$ disjoint Hamiltonian cycles of G. Theorem 1 is proved.

3. Proof of Conjecture 1 for $k = 1$

A proof of conjecture 1 for $k = 1$ is contained in reference [6]. Here we use the method posed in section 2 to simplify that proof. For explicitness, we rewrite Conjecture 1 for $k = 1$ as the following Theorem.

Theorem 2 Let G be a 4-connected graph. If $uv \notin E(G) \Longrightarrow \max\{d(u), d(v)\} \geq \frac{n}{2} + 2$, then G contains two disjoint Hamiltonian cycles.

Proof Let U_1, V_1, W_1 and S_1 be defined as in section 2. That $k = 1$ implies $|U_1 \cup W_1| = |V_1 \cup W_1| = 4$, and so $0 \leq |\bar{E}(V_1, W_1)| \leq 2$. We just consider the case $|\bar{E}(V_1, W_1)| = 2$ since the other cases are simpler. Clearly, $|V_1| \geq 2$. Let $v_1w_1, v_2w_2 \in \bar{E}(V_1, W_1)$. Then $v_1 \neq v_2$.

If $w_1 \neq w_2$, then $C_1 = P_1Q_1$ where $Q_1 = v_1w_1w_2v_2$ is a Hamiltonian cycle of G^*, but not of G, and $C_2 = P_2Q_2$ where $Q_2 = w_1v_2v_1w_2$ is a Hamiltonian cycle of G. By Lemma 7, we can construct a Hamiltonian cycle of G from the cycle C_1 of G^*. Let $s \in P_1 \setminus \{u_1\}, t \in P_1 \setminus \{u_2\}$ be two vertices adjacent to v_1 and v_2, respectively. By Lemma 7, $\{s, t\} \neq \{u_1, u_2\}$, say $s \neq u_2$. If t lies in $P(s, u_2)$, we substitute $v_1s, w_1s^+_{C_1}$ for $ss^+_{C_1}, v_1w_1$, and $v_2t, w_2t^-_{C_1}$ for $tt^-_{C_1}, v_2w_2$ to get a Hamiltonian cycle in $G - E(C_2)$. If t lies in $P(u_1, s^+_{C_1})$, we substitute $v_1s, v_2t, w_2t^-_{C_1}$ and $w_1s^-_{C_1}$ for $v_1w_1, v_2w_2, ss^-_{C_1}$ and $tt^-_{C_1}$ to get a Hamiltonian cycle in $G - E(C_2)$.

Conseqently, we assume that $w_1 = w_2 = w$. Then $V_1 = \{v_1, v_2, v_3\}$. Let $v_1w, v_2w \in \bar{E}(V_1, W_1)$. Let $Q_1 = v_1v_3wv_2$ and $Q_2 = v_3v_2v_1w$. Then $C_1 = P_1Q_1$

and $C_2 = P_2Q_2$ are two disjoint Hamiltonian cycles of G^*. Let $s \in P_2$ be a neighbour of v_1.

If $s \neq u_3$,then we can get two disjoint Hamiltonian cycles of G in the same way as above.

If $s = u_3$, then $C_2' = C_2 + \{wv_3, v_1u_3\} - \{u_3v_3, wv_1\}$ is a Hamiltonian cycle of G. Let $C_1' = C_1 - \{v_3w\}$. By Lemma 7, there is a vertex $t \in S \setminus (U_1 \cup W_1)$, such that $tv_2 \in E(G)$ or $tv_3 \in E(G)$.

If $tv_2 \in E(G)$, substituting u_3w, u_3v_3 for wv_3, and $u_3^+u_3^-$ for $u_3^+u_3u_3^-$, and $v_2t, wt_{C_1}^-$ for $wv_2, tt_{C_1}^-$(if $t = u_3^+$ then $t_{C_1}^-$ should be taken as u_3^-), we get a Hamiltonian cycle C_1'' with the only one edge $u_3^+u_3^-$ which may be a common edge of C_1'' and C_2'. If $u_3^+u_3^- \notin E(C_2')$, the two disjoint Hamiltonian cycles of G are obtained. Otherwise, by Lemma 6, we have

$$\begin{aligned} & d_{G-E(C_2')}(u_3^+) + d_{G-E(C_2')}(u_3^-) \\ = {} & 2(n-l-2-1) \geq 2(\frac{n}{2} + 2k + l + 1 - l - 2 - 1) \\ = {} & n + 4k - 4 \geq n. \end{aligned}$$

By the method used to prove Lemma 1 (2), we can show that $G - E(C_2')$ contains a Hamiltonian cycle, and so G contains two disjoint Hamiltonian cycles.

If $tv_2 \notin E(G)$, then $tv_3 \in E(G)$, and $v_2u_1 \in E(G)$ or $v_2u_3 \in E(G)$ (say $v_2u_1 \in E(G)$). Substituting v_3tw for v_3w, and $t_{C_1}^+t_{C_1}^-$ for $t_{C_1}^-tt_{C_1}^+$, and u_1v_2, u_1^-w (if $u_1 = t_{C_1}^+$, then u_1^- should be taken as $t_{C_1}^-$) for v_2w and $u_1u_1^-$, we get a Hamiltonian cycle C_1'' with only the one edge $t_{C_1}^-t_{C_1}^+$ which may be a common edge of C_1'' and C_2'. Note that $t_{C_1}^-, t_{C_1}^+ \in S$. It follows from the discussion above that G contains two disjoint Hamiltonian cycles of G. The proof of Theorem 2 is completed.

References

[1] L.W.Beineke and R.L.Wilson, Selected topics in graph theory, Academic Press, New York(1978)161.

[2] J.A.Bondy and U.S.R.Murty, Graph Theory with Applications, The Macmillan Press, London(1976).

[3] G.H.Fan, New sufficient condition for cycles in graphs, J.Combin. Theory B37(1984)221-227.

[4] H.Li, Edge disjoint cycles in graphs, J.Graph Theory 13(1989), No.3, 313-322.

[5] F.Tian, A short proof of a theorem about the circumference of a graph, J.Combin. Theory B45(1988)373-375.

[6] S.Zhou, Disjoint Hamiltonian cycles in Fan $2k$-type graphs, J. Graph Theory 3(1989), 313-322.

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