# On Finite Graphs that are Self-complementary and Vertex-transitive*

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### Abstract

Self-complementary graphs have been extensively investigated and effectively used in the study of Ramsey numbers. It is well-known and easily proved that there exist regular self-complementary graphs of order $n$ if and only if $n \equiv 1 \pmod 4$. So it is natural to ask whether a similar result for self-complementary vertex-transitive graphs exists. More precisely, for which positive integers $n$ do there exist self-complementary vertex-transitive graphs of order $n$? Recently, several long-standing open problems concerning this question have been solved, but the general question is still open. In this article we give a brief survey in this area and introduce several new problems.

## 1  Introduction

The complement $\overline{\Gamma}$ of a graph $\Gamma$ is the graph with vertex set $V\Gamma$ such that two vertices $x, y$ are adjacent if and only if $x, y$ are not adjacent in $\Gamma$. A graph is said to be *self-complementary* if it is isomorphic to its complement. The study of self-complementary graphs has a rich history (see for instance [10, 11, 14, 22, 26, 28, 31]). Various enumeration and constructive results for such graphs have been obtained (see for example [11, 13, 28]), and various graph theoretic properties of such graphs have been investigated (see for example [8, 27, 33]). In a remarkable paper [21], L. Lovász proved that the *Shannon capacity* of a vertex-transitive self-complementary graph of order $n$ is equal to $\sqrt{n}$. Also this class of graphs has been used to study Ramsey numbers (see for instance [10, 12, 16, 29]).

A graph $\Gamma$ is said to be *vertex-transitive* if $\operatorname{Aut}\Gamma$ is transitive on $V\Gamma$. For a finite group $G$, let $G^{\#} = G \setminus \{1\}$, the set of all non-identity elements of $G$. For a subset $S \subseteq G^{\#}$, the *Cayley digraph* $\operatorname{Cay}(G, S)$ is the digraph $\Gamma$ with vertex set $G$ such that

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$(x, y)$ is an edge of $\Gamma$ if and only if $yx^{-1} \in S$. If $S = S^{-1} := \{s^{-1} \mid s \in S\}$, then $\text{Cay}(G, S)$ may be viewed as an undirected graph, and if $G$ is cyclic then $\text{Cay}(G, S)$ is a *circulant*. It is easily checked that the group $G$ acting by right multiplication (that is, $g : x \to xg$) is a subgroup of $\text{Aut}\,\Gamma$ and acts regularly on $V\Gamma = G$. In particular, Cayley graphs are vertex-transitive. However, the converse is not necessarily true (see, for example, [23]). The objective of this article is the study of graphs that are both self-complementary and vertex-transitive.

One of the principal motivations for studying self-complementary vertex-transitive graphs is to investigate Ramsey numbers. In the study of diagonal Ramsey numbers, one wishes to construct a 'big' graph $\Gamma$ such that neither $\Gamma$ nor $\overline{\Gamma}$ contains $K_m$ for a given $m$ so that $\Gamma$ and $\overline{\Gamma}$ should not be very different. Moreover, as $K_m \not< \Gamma$ is a global property of $\Gamma$, the edges of $\Gamma$ should be distributed 'homogeneously'. In the extreme case, $\Gamma$ is self-complementary and vertex-transitive. This is one of the reasons why more constructions of self-complementary vertex-transitive graphs is desirable.

Recently, several long-standing open questions regarding self-complementary vertex-transitive graphs have been solved, and some new open problems have naturally arisen. In this paper, we shall give a brief survey on this topic. For convenience, in the ensuing sections we shall call a graph $\Gamma$ an *SCVT-graph* if $\Gamma$ is self-complementary and vertex-transitive.

# 2   Constructing SCVT-graphs

It follows from the definition that if

$$\Gamma = \text{Cay}(G, S), \text{ and } S^\sigma = G^\# \setminus S \text{ for some } \sigma \in \text{Aut}(G), \tag{1}$$

then $\Gamma$ is self-complementary. We shall see that this property can produce many SCVT-graphs. In fact, all known constructions of SCVT-graphs are based on this property. For brevity, we shall say a group $G$ and $\sigma \in \text{Aut}(G)$ with this property is an *R-group* and an *R-automorphism*, respectively (R stands for *Reflexible*, refer to [14, p. 627]). We shall also call such a graph $\text{Cay}(G, S)$ an *SC-CI-graph* of $G$ (where CI stands for *Cayley Isomorphism*, which comes from the isomorphism problem for Cayley graphs, refer to [2]).

In [32], all reflexible abelian groups are characterized, and non-abelian groups of order prime-cube are investigated. It was proved in [3] that for an R-automorphism $\sigma$, $\langle \sigma^2 \rangle$ has $2m$ orbits $\Delta_1, \Delta_2, \ldots, \Delta_{2m}$ on $G^\#$ for some positive integer $m$ such that $\Delta_i^\sigma = \Delta_{i+1}$ for all odd $i$. Using these simple properties, a method of constructing SCVT-graphs can be described as follows (see [3]).

**Construction 2.1** Let $\mathcal{C}(G, \sigma)$ be the class of the graphs which are constructed using the following steps:

  (i) find an R-group $G$ and an R-automorphism $\sigma$ of $G$;

  (ii) find $\langle \sigma^2 \rangle$-orbits $\Delta_1, \Delta_2, \ldots, \Delta_{2m}$ on $G^\#$ such that $\Delta_i^\sigma = \Delta_{i+1}$ for all odd $i$;

(iii) set $S = S(\varepsilon_1, \varepsilon_3, \ldots, \varepsilon_{2m-1}) = \bigcup_{\text{odd } i} \Delta_i^{\sigma^{\varepsilon_i}}$ where $\varepsilon_i = 0$ or $1$, and set $\Gamma = \text{Cay}(G, S)$.

Since $(\Delta_i^{\sigma^{\varepsilon_i}})^\sigma = \Delta_i^{\sigma^{1-\varepsilon_i}}$, we have $S^\sigma = \bigcup_{\text{odd } i} \Delta_i^{\sigma^{1-\varepsilon_i}} = G^\# \setminus S$ so that $\Gamma$ is self-complementary. Together with the next simple proposition, Construction 2.1 can produce various SCVT-graphs. An automorphism $\sigma$ of a group $G$ will be said to be *fixed-point free* if $\sigma$ fixes no element of $G^\#$.

**Proposition 2.2** ([3, Proposition 2.3]) *Suppose that $G$ is a finite group of odd order.*

(1) *Let $\sigma \in \text{Aut}(G)$ be such that $\sigma^2$ is fixed-point free and each $g \in G$ is conjugate under $\langle \sigma^2 \rangle$ to $g^{-1}$. Then $G$ is an R-group and $\sigma$ is an R-automorphism of $G$.*

(2) *Let $G = G_1 \times G_2 \times \ldots \times G_s$ be such that each $G_i$ is an R-group and $\sigma_i$ is an R-automorphism of $G_i$ of order 4. Then $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_s) \in \text{Aut}(G)$ is an R-automorphism of $G$.*

The next are some examples of constructing SCVT-graphs.

**Example 2.3** (Sachs [31]) Let $p$ be a prime such that 4 divides $p-1$, and let $G \cong \mathbb{Z}_{p^d}$ where $d \geq 1$. Then $\text{Aut}(G) \cong \mathbb{Z}_{p^{d-1}(p-1)}$, and by Proposition 2.2 (1), each $\sigma \in \text{Aut}(G)$ of order divisible by 4 is an R-automorphism of $G$. Therefore, by Construction 2.1 and Proposition 2.2, if $n = p_1^{d_1} p_2^{d_2} \ldots p_s^{d_s}$ such that 4 divides $p_i - 1$ for each $i$ then there exist circulant SC-CI-graphs of order $n$.

**Example 2.4** (see Zelinka (1979), Suprunenko (1985) or Rao (1985)) Let $p$ be a prime, and let $G \cong \mathbb{Z}_p^d$ such that 4 divides $p^d - 1$. Then $\text{GL}(1, p^d) < \text{Aut}(G)$, and by Proposition 2.2 (1), each $\sigma \in \text{GL}(1, p^d)$ of order divisible by 4 is an R-automorphism of $G$. Therefore, by Construction 2.1 and Proposition 2.2, if $n = p_1^{d_1} p_2^{d_2} \ldots p_s^{d_s}$ such that 4 divides $p_i^{d_i} - 1$ for all $i$ then there exist SC-CI-graphs of order $n$.

The graphs given in Examples 2.3 and 2.4 are all Cayley graphs on abelian groups. The first family of self-complementary Cayley graphs on non-nilpotent groups was constructed in [3] as follows.

**Proposition 2.5** *Let $p$ be an odd prime and let $q$ be a primitive prime of $p^4 - 1$. Let $G = \mathbb{Z}_p^4 \rtimes \mathbb{Z}_q \leq \text{AGL}(1, p^4)$. Then there exist Cayley graphs on $G$ which are self-complementary.*

# 3   Self-complementary circulants

By Example 2.3, there exist self-complementary circulants of order $n$ for all $n$ each of whose prime divisors is congruent to 1 modulo 4. H. Sachs (1962) posed a conjecture that only for such $n$ do there exist self-complementary circulants of order $n$. The following theorem is a solution of this conjecture.

**Theorem 3.1** (Fronček, Rosa and Širáň (1996)) *For a positive integer $n$, there exist self-complementary circulants of order $n$ if and only if all prime divisors of $n$ are congruent to 1 modulo 4.*

The proof of the theorem in [14] is by counting the so-called *closed walks*, which is graph theoretic. In [4], a group theoretic proof is given, which is based on the well-known result of Burnside and Schur that the cyclic groups of composite order are *Burnside groups* (see [34, Theorem 25.3]).

Therefore, for all possible orders $n$, Construction 2.1 can produce some self-complementary circulants of order $n$. Further, by Muzychuk (1995), Construction 2.1 can produce all self-complementary circulants of square-free orders. The special cases of orders $p$ and $pq$ ($p, q$ are distinct primes) are well studied, and enumeration results for self-complementary circulant digraphs of such orders are obtained using Polya's theorem in [6, 7, 9], also refer to B. Alspach's review for [7]. Here we only mention a simple consequence of Construction 2.1, which is given in [3].

**Corollary 3.2** *Let $p$ be a prime such that 4 divides $p - 1$, and write $p - 1 = 2^r t$ such that $t$ is odd. Then there are*

$$\sum_{2 \leq k \leq r} 2^{(p-1)/2^k}$$

*SCVT-graphs of order $p$.*

For self-complementary circulants of prime-power order, a characterization result is obtained in [3], which is based on Construction 2.1 and the wreath product of graphs. The *wreath product* $\Gamma_1 \wr \Gamma_2$ of two graphs $\Gamma_1 = (V_1, E_1)$ and $\Gamma_2 = (V_2, E_2)$ is the graph with vertex set $V_1 \times V_2$ such that $\{(a_1, a_2), (b_1, b_2)\}$ is an edge if and only if either $\{a_1, b_1\} \in E_1$ or $a_1 = b_1$ and $\{a_2, b_2\} \in E_2$.

**Theorem 3.3** ([3, Theorem 1.5]) *Let $\Gamma$ be a self-complementary circulant of order $p^n$ for some prime $p$ and some $n \geq 2$. Then one of the following holds:*

(i) $\Gamma \in \mathcal{C}(\mathbb{Z}_{p^n}, \sigma)$ *for some $\sigma \in \mathrm{Aut}(\mathbb{Z}_{p^n})$ whose order divides $p - 1$;*

(ii) $\Gamma = \Gamma_0 \wr \Gamma_1$, *where $\Gamma_0$ and $\Gamma_1$ are self-complementary circulants of order $p^{n_0}$ and $p^{n_1}$, respectively, such that $n_0 + n_1 = n$.*

# 4  The existence problem

It is well-known and easily proved that there exists a self-complementary graph of order $n$ if and only if $n$ is congruent to 0 or 1 modulo 4, and that there exists a regular self-complementary graph of order $n$ if and only if $n$ is congruent to 1 modulo 4.

Since vertex-transitive graphs are regular, the following natural question was proposed by Zelinka (1979).

**Question 4.1** Is it true that for every $n \equiv 1 \pmod 4$ there exists an SCVT-graph of order $n$?

In [14, Section 4], Fronček, Rosa and Širáň in [14] asked the same question for a product of two distinct primes $p$ and $q$, that is, whether there exist SCVT-graphs of order $pq$ where $p$ and $q$ are congruent to 3 (mod 4)? These questions had been open until very recently when a negative answer was given by the author in [19] as follows.

**Theorem 4.2** Let $n = pq$ where $p, q$ are distinct primes. Then there exist SCVT-graphs of order $n$ if and only if $p, q \equiv 1 \pmod 4$.

The proof of this theorem in [19] was naturally divided into two steps, and depends on the following lemma.

**Lemma 4.3** Let $\Gamma$ be an SCVT-graph. Let $G$ be a subgroup of $\mathrm{Aut}\,\Gamma$ which is transitive on $V\Gamma$. Suppose that $G$ is of rank $r$ and has subdegrees $d_0, d_1, d_2, \ldots, d_{r-2}, d_{r-1}$ which satisfy $d_i \le d_{i+1}$ for all $i \in \{0, 1, \ldots, r-2\}$. Then $r$ is odd, $d_0 = 1$ and $d_{i+1} = d_i$ for every odd $i \in \{1, 2, \ldots, r-2\}$.

The first step excludes the vertex-primitive case by checking the classification of vertex-primitive graphs of order $pq$ obtained by Praeger and Xu in [25]. We found that there do not exist vertex-primitive graphs of order $pq$ which are self-complementary. The second step treats the vertex-imprimitive case by induction on the graph orders. We notice that the classification of vertex-primitive graphs of order $pq$ of Praeger and Xu is dependent on the classification of finite simple groups, and so is Theorem 4.2.

Therefore, by Theorem 4.2, there do not exist SCVT-graphs of order $n$ for each $n \equiv 1 \pmod 4$. The existence problem for self-complementary vertex-transitive or Cayley graphs naturally arises here (refer to [14, Section 4]).

**Problem 4.4** Determine positive integers $n$ such that there exist self-complementary vertex-transitive or Cayley graphs of order $n$.

Let $SCVT$ and $SCCay$ denote the set of positive integers $n$ for which there exist SCVT-graphs and self-complementary Cayley graphs of order $n$, respectively. Note that Cayley graphs are vertex-transitive. By Example 2.4 and Theorem 4.2, we have

$$C \subseteq SCCay \subseteq SCVT \subset D$$

where $C = \{p_1^{d_1} \ldots p_r^{d_r} \mid p_i^{d_i} \equiv 1 \pmod 4\}$ and $D = \{n \mid n \equiv 1 \pmod 4\}$.

For Cayley graphs, Fronček, Rosa and Širáň in [14] proved

**Theorem 4.5** Let $n = ps$, where $p \equiv 3 \pmod 4$ is a prime and $p > s$. Then no Cayley graphs of order $n$ are self-complementary.

The general case for Problem 4.4 seems very hard. To approach it, we are particularly interested in the following extreme cases (see [19, Section 3]).

**Question 4.6**

(1) Does $SCCay = C$, in others words, do there exist self-complementary Cayley graphs of order $n$ with $n \notin C$?

(2) Does $SCVT = SCCay$, in other words, do there exist self-complementary vertex-transitive graphs which are not Cayley graphs?

# 5 SC-CI-graphs

In Section 2, several families of SCVT-graphs are produced by Construction 2.1, which are all SC-CI-graphs. The question of whether or not every self-complementary Cayley graph $\mathrm{Cay}(G, S)$ is an SC-CI-graph of $G$ had been open until very recently when a negative answer was given by R. Jajcay and the author in [17] by producing infinite families of counterexamples. The counterexamples in [17] are constructed using the wreath product of certain self-complementary Cayley graphs.

**Theorem 5.1** *Let $p$ be a prime such that 4 divides $p - 1$, and let $\sigma_1, \sigma_2 \in \mathrm{Aut}(\mathbb{Z}_p)$ such that 4 divides both $o(\sigma_1)$ and $o(\sigma_2)$. Set $\Gamma_i = \mathrm{Cay}(\mathbb{Z}_p, \sigma_i)$ with $i = 1, 2$, and $\Gamma = \Gamma_1 \wr \Gamma_2$. Then $\Gamma$ is a Cayley graph of $\mathbb{Z}_{p^2}$ which is self-complementary, and further $\Gamma$ is an SC-CI-graph of $\mathbb{Z}_{p^2}$ if and only if $o(\sigma_1) = o(\sigma_2)$. In particular, if 8 divides $p - 1$ then there exist self-complementary Cayley graphs $\mathrm{Cay}(\mathbb{Z}_{p^2}, S)$ which are not SC-CI-graphs of $\mathbb{Z}_{p^2}$.*

Theorem 5.1 provides self-complementary Cayley graphs $\Gamma$ on $\mathbb{Z}_{p^2}$ which are not SC-CI-graphs of $\mathbb{Z}_{p^2}$. The following theorem characterizes self-complementary graphs of prime-square order.

**Theorem 5.2** ([3]) *Let $\Gamma$ be an SCVT-graph of order $p^2$ where $p$ is a prime. Then one of the following holds:*

(i) $\Gamma = \Gamma_0 \wr \Gamma_1$ *where $\Gamma_i \in \mathcal{C}(\mathbb{Z}_p, \sigma_i)$ for some $\sigma_i \in \mathrm{Aut}(\mathbb{Z}_p)$;*

(ii) $\Gamma \in \mathcal{C}(\mathbb{Z}_{p^2}, \sigma)$ *for some $\sigma \in \mathrm{Aut}(\mathbb{Z}_{p^2})$ such that $o(\sigma)$ divides $p - 1$;*

(iii) $\Gamma \in \mathcal{C}(\mathbb{Z}_p^2, \sigma)$ *for some $\sigma \in \mathrm{GL}(2, p)$ such that $o(\sigma)$ divides $p - 1$ or $p + 1$.*

By results of [18, 24], the graph $\Gamma = \Gamma_0 \wr \Gamma_1$ in part (i) is also a Cayley graph on $\mathbb{Z}_p^2$, and therefore, by Godsil [15], $\Gamma$ is an SC-CI-graph of $\mathbb{Z}_p^2$. Actually, we still do not know whether there exists a self-complementary Cayley graph which cannot be represented as an SC-CI-graph of any group.

By Example 2.4, if $n \in C$ then SCVT-graphs of orders $n$ can be constructed by Construction 2.1. Regarding Problem 4.4, naturally we wish to use Construction 2.1 to construct SCVT-graphs of other orders. Unfortunately, the following result shows that this is not possible, and demonstrates how hard it is to construct SCVT-graphs of order $n$ for $n \notin C$ (if they exist).

**Theorem 5.3** ([3, Theorem 1.3]) *Let* $\Gamma$ *be a self-complementary graph of order* $n$. *Then* $\Gamma$ *may be represented as an SC-CI-graph of some group if and only if* $n \in C$.

Therefore, using only Construction 2.1 and the wreath product, we cannot construct SCVT-graphs of order $n$ for $n \notin C$.

# 6 The vertex-primitive case

In the final section, we discuss self-complementary vertex-primitive graphs. In Example 2.4, taking $\sigma$ to be of order $(p^d - 1)/2$, the associated SC-CI-graphs of order $p^d$ are the *Paley graphs*. It easily follows that the Paley graphs are arc-transitive. Moreover, H. Zhang (1992) proved that all self-complementary edge-transitive graphs are exactly the Paley graphs. In [37], Zhang also obtained a similar result for self-complementary edge-transitive digraphs.

We notice that Paley graphs are vertex-primitive, and moreover, they are the only known examples of self-complementary vertex-primitive graphs. Let $\Gamma$ be a vertex-primitive graph of order $n$ which is self-complementary. Then $n \equiv 1 \pmod 4$, in particular, $n$ is odd. Thus Aut $\Gamma$ is a primitive permutation group of odd degree, and such groups are classified by Liebeck and Saxl in [20]. Hence a reasonable approach to Problem 4.4 is to search for a complete solution of the following problem.

**Problem 6.1** ([19, Problem 3.3]) Classify self-complementary vertex-primitive graphs.

From the proof of Theorem 4.2 in [19], we know that there do not exist self-complementary vertex-primitive graphs of order a product of two distinct primes. We are inclined to think that the only possible orders of self-complementary vertex-primitive graphs are prime powers.

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